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COMMON FIXED POINT THEOREMS FOR SIX WEAKLY COMPATIBLE SELF-MAPPINGS IN *M*- FUZZY METRIC SPACES

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***Abstract*—** In this paper we define *M* - fuzzy metric space and prove a common fixed point theorem for six weakly compatible self-mappings in complete *M*- fuzzy metric space.

***Keywords*—** *M* - Fuzzy Metric Space, Weakly Compatible Self-mappings, Coincidence points, Hadzic-Type *t*-norm.

1. Introduction

The concept of fuzzy sets was introduced initially by Zadeh [39], in 1965. Since then many authors have used this concept expansively and developed the theory of fuzzy sets and applications especially, Deng [2]

Erceg [6], George and Veeramani [8] have introduced fuzzy metric which was used in topology and analysis. Recently Gregori et. al [10, 12] and Rafi et. al [28] have studied some property in fuzzy metric spaces. Many authors [1, 6-8, 9, 11, 13, 14, 17, 18, 25, 26, 27-33, 37, 38] have studied fixed point theory in fuzzy metric spaces.

In this paper we define *M*- fuzzy metric space and prove a common fixed point theorem for six weakly compatible self-mappings in complete *M*- fuzzy metric spaces.

2. Preliminaries

2.1 Coincidence Point

Let f, g be self-mappings on a non-empty set X. A point $x \in X$ is said to be coincidence point of f and g if $f(x) = g(x)$. □

2.2 Weakly Compatible Mappings

Two self-mappings f and g of a non-empty set X are said to be weakly compatible (or coincidentally commuting) if they commute at their coincidence points, i.e., if $fz = gz$ for some $z \in X$, then $fgz = gfz$.

2.3 Hadzic-Type *t*-norm

Let T be a t-norm and $T^n : [0, 1] \to [0, 1]$ be defined by $T^1(x) = T(x)$, $T^{n+1}(x) = T(T^n(x), x)$, for all $n \in N$, and $x \in (0, 1)$.

Then we say that the t-norm T is of Hadzic-Type if the family $\{T^n(x);\ n \in N\}$ is equicontinuous at $x = 1$ if for every $\lambda \in (0, 1)$ there exists $\delta(\lambda) \in (0, 1)$ such that

$$x > 1 - \delta(\lambda) \Rightarrow T^{n}(x) > 1 - \lambda.$$

2.4 *M*- Fuzzy Metric Space

A 3-tuple (X, M, T) is called a M- fuzzy metric space if X is a non-empty set, T is continuous t-norm, and M is a fuzzy set on $X^3 \times (0, \infty)$ satisfying the following conditions: for all $x, y, z, a \in X$ and $t, s > 0$,

(FM-1) $M(x, y, z, t) > 0$,

(FM-2) $M(x, y, z, t) = 1$ if and only if $x = y = z$,

(FM-3) $M(x, y, z, t) = M(p\{x, y, z\}, t)$ (symmetry), where p is a permutation function

(FM-4) $T(M(x, y, a, t), M(a, z, z, s)) \leq M(x, y, z, t + s)$,

(FM-5) $M(x, y, z, \cdot): (0, \infty) \rightarrow [0, 1]$ is continuous.

3. Common Fixed Point Theorem for Six Weakly Compatible Self- Mappings in *M* -Fuzzy Metric Space

3.1 *Definition*

Let (X, M, T) be a M -fuzzy metric space. Then M is said to

be continuous function on $X^3 \times (0, \infty)$ if

$$\lim_{n\to\infty} M(x_n, y_n, z_n, t_n) = M(x, y, z, t)$$

whenever a sequence $\{(x_n, y_n, z_n, t_n)\}$ in $X^3 \times (0, \infty)$ converges to a

point $(x, y, z, t) \in X^3 \times (0, \infty)$, i.e.,

$$\lim_{n\to\infty} x_n = x, \quad \lim_{n\to\infty} y_n = y, \quad \lim_{n\to\infty} z_n = z, \quad \lim_{n\to\infty} t_n = t$$

and

$$\lim_{n\to\infty} M(x_n, y_n, z_n, t_n) = M(x, y, z, t). \quad \square$$

3.2 Lemma

Let (X, M, T) be a M -fuzzy metric space. Then M is a continuous function on $X^3 \times (0, \infty)$.

Proof :

Let $x, y, z \in X$ and $t > 0$, and let $\{(x_n', y_n', z_n', t_n')\}$ be a sequence in $X^3 \times (0, \infty)$ that converges to (x, y, z, t). Since $\{M(x_n', y_n', z_n', t_n')\}$

is a sequence in $(0, 1]$, there is a subsequence $\{(x_n, y_n, z_n, t_n)\}$ of the sequence $\{(x_n', y_n', z_n', t_n')\}$ such that the sequence $\{M(x_n, y_n, z_n, t_n)\}$ converges to some point of [0, 1]. Fix $\delta > 0$ such that $\delta < \frac{t}{2}$. Then there is an $n_0 \in \mathbb{N}$ such that $|t - t_n| < \delta$ for all $n \geq n_0$. Hence we have

$$M(x_n, y_n, z_n, t_n)$$

$$\geq M(x_n, y_n, z_n, t - \delta)$$

$$\geq T\left(M\left(x_n, y_n, z, t - \frac{4\delta}{3}\right), M\left(z, z_n, z_n, \frac{\delta}{3}\right)\right)$$

$$\geq T^2\left(M\left(x_n, z, y, t - \frac{5\delta}{3}\right), M\left(y, y_n, y_n, \frac{\delta}{3}\right), M\left(z, z_n, z_n, \frac{\delta}{3}\right)\right)$$

$$\geq T^3\left(M(z, y, x, t - 2\delta), M\left(x, x_n, x_n, \frac{\delta}{3}\right), M\left(y, y_n, y_n, \frac{\delta}{3}\right), M\left(z, z_n, z_n, \frac{\delta}{3}\right)\right)$$

and

$$M(x, y, z, t + 2\delta)$$

$$\geq M(x, y, z, t_n + \delta)$$

$$\geq T\left(M\left(x, y, z_n, t_n + \frac{2\delta}{3}\right), M\left(z_n, z, z, \frac{\delta}{3}\right)\right.$$

$$\geq T^{2}\left(M\left(x, z_n, y_n, t_n + \frac{\delta}{3}\right), M\left(y_n, y, y, \frac{\delta}{3}\right), M\left(z_n, z, z, \frac{\delta}{3}\right)\right)^{3}$$

$$\geq T^{3}\left(M(z_n, y_n, x_n, t_n), M\left(x_n, x, x, \frac{\delta}{3}\right), M\left(y_n, y, y, \frac{\delta}{3}\right), M\left(z_n, z, z, \frac{\delta}{3}\right)\right)$$

for all $n \geq n_0$. Letting $n \to \infty$, we obtain

$$\lim_{n\to\infty} M(x_n, y_n, z_n, t_n) \geq T^{3}(M(x, y, z, t - 2\delta), 1, 1, 1)$$

$$= M(x, y, z, t - 2\delta)$$

and

$$M(x, y, z, t + 2\delta) \geq \lim_{n\to\infty} T^{3}(M(x_n, y_n, z_n, t_n), 1, 1, 1)$$

$$= \lim_{n\to\infty} M(x_n, y_n, z_n, t_n)$$

respectively. So, by continuity of the function $t \to M(x, y, z, t)$, we immediately deduce that

$$\lim_{n\to\infty} M(x_n, y_n, z_n, t_n) = M(x, y, z, t).$$

Therefore is continuous on $X^{3} \times (0, \infty)$. □

3.3 *Definition*

Let f and r be mappings from a M-fuzzy metric space (X, M, T) into itself. Then the mappings f and r are said to be

(1) Weakly compatible if they commute at a coincidence point, that is,

$$fx = rx \text{ implies } frx = rfx.$$

(2) Compatible if for all $t > 0$,

$$\lim_{n\to\infty} M(frx_n, rfx_n, rfx_n, t) = 1$$

whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} fxn = \lim_{n\to\infty} rxn = x$$

for some $x \in X$. □

3.4 Lemma

Let (X, M, T) be a M-fuzzy metric space. If a sequence $\{x_n\}$

in X exists such that for every $n \in$ `, $0 < k < 1$ and $t > 0$

$$M(x_n, x_n, x_{n+1}, k^n t) \geq M(x_0, x_0, x_1, t)$$

then the sequence $\{x_n\}$ is a Cauchy sequence.

***Proof* :**

Since t-norm T is of Hadzic-type, we have for each $\varepsilon \in (0, 1)$ there

exists a $\delta \in (0, 1)$ such that

$$x > 1 - \delta \Rightarrow T^n(x) > 1 - \varepsilon, \forall\ n \geq 1.$$

Since, $\lim_{t\to\infty} M(x_0, x_0, x_1, t) = 1$, there exists $t_0 > 0$ such that

$$M(x_0, x_0, x_1, t_0) > 1 - \delta \quad \text{and } T^n(M(x_0, x_0, x_1, t_0)) > 1 - \varepsilon, \forall\ n \geq 1.$$

Since $\sum_{n=0}^{\infty} k^n t_0 < \infty$, we have for every $t > 0$ there exists an n_0 such that for $n \geq n_0$ we have,

$$\sum_{i=n}^{\infty} k^i t_0 < t.$$

Thus for every $n \geq n_0$ and $\forall\ m$,

$$M(x_n, x_n, x_{n+m+1}, t) \geq M\left(x_n, x_n, x_{n+m+1}, \sum_{i=n}^{\infty} k^i t_0\right)$$

$$\geq M\left(x_n, x_n, x_{n+m+1}, \sum_{i=n}^{n+m} k^i t_0\right)$$

$$\geq T_{i=n}^{n+m} M(x_i, x_i, x_{i+1}, k^i t_0)$$

$$\geq T_{i=0}^{m} M(x_{i+n}, x_{i+n}, x_{i+n+1}, k^{i+n} t_0)$$

$$\geq T^m M(x_0, x_0, x_1, t_0)$$

$$> 1 - \varepsilon.$$

Hence the sequence $\{x_n\}$ is Cauchy. □

Now we prove our main result of this paper.

3.5 *Theorem*

Let f, g, h, p, q and r be self-mappings of a fuzzy metric space (X, M, T) satisfying:

(i) $p(X) \subseteq hr(X)$, $q(X) \subseteq fg(X)$ subset of X

(ii) $hr(X)$ or $fg(X)$ is a closed

(iii) The pairs (q, hr) and (p, fg) are weakly compatible and

$hr = rh$, $gh = hg$, $qr = rq$ and $fg = gf$

(iv) $M(px, qy, qy, kt) \times$
$T(M(px, qy, qy, kt), M(fgx, px, px, kt)) \times M(hry, qy, qy, kt)$

$$\geq a(t)(M(fgx, px, px, t) + b(t)M(fgx, hry, hry, t)$$
$$\times M(hry, qy, qy, kt)$$

for every $x, y \in X$, and $t > 0$ and for some $k \in (0, 1)$, where $a, b : \mathbb{R}^+ \to (0, 1]$ be two functions such that $a(t) + b(t) = 1$.

Then, f, g, h, p, q and r have a unique common fixed point in X.

***Proof* :**

Let $x_0 \in X$ be an arbitrary point. By (i), there exist $x_1, x_2 \in X$ such that

$$px_0 = hrx_1 = y_0 \text{ and } qx_1 = fgx_2 = y_1.$$

Inductively construct a sequence $\{y_n\}$ in X such that

$$y_{2n} = px_{2n} = hrx_{2n+1} \text{ and } y_{2n+1} = fgx_{2n+2} = qx_{2n+1}, n = 0, 1, 2, \ .$$

Now, we prove that the sequence $\{y_n\}$ is Cauchy. Let

$$d_m(t) = M(y_m, y_{m+1}, y_{m+1}, t).$$

Then, putting $x = x_{2n}$, $y = x_{2n+1}$ in (iii), we have

$M(px_{2n}, qx_{2n+1}, qx_{2n+1}, kt)\times$

$T(M(px2n, qx2n+1, qx2n+1, kt), M \quad (fgx_{2n}, px_{2n}, px_{2n}, \quad kt))$

$\times M(hrx_{2n+1}, qx_{2n+1}, qx_{2n+1}, \quad kt)$

$\geq (a(t)M(fgx_{2n}, px_{2n}, px_{2n}, t) + b(t)M(fgx_{2n}, hrx_{2n+1}, hrx_{2n+1}, t))$

$\times M(fgx_{2n}, qx_{2n+1}, qx_{2n+1}, 2kt).$

Thus

$M(y_{2n}, y_{2n+1}, y_{2n+1}, kt)\times$

$T(M(y2n, y2n+1, y2n+1, kt), M(y2n-1, y2n, y2n, kt))$

$\times \quad M(y_{2n}, y_{2n+1}, y_{2n+1}, kt)$

$\geq (a(t)M(y2n-1, y2n, y2n, t) + b(t)M(y2n-1, y2n, y2n, t))$

$\times M(y_{2n-1}, y_{2n+1}, y_{2n+1}, 2kt).$

Hence

$d_{2n}(kt)M(y_{2n-1}, y_{2n+1}, y_{2n+1}, 2kt)$

$\geq a(t)d_{2n-1}(t) + b(t)d_{2n-1}(t)\ M(y_{2n-1}, y_{2n+1}, y_{2n+1}, 2kt).$

Thus

$d_{2n}(kt) \geq d_{2n-1}(t)$. Putting $x = x_{2n+2}$, $y = x_{2n+1}$ in (iii) we have

$M(px_{2n+2}, qx_{2n+1}, qx_{2n+1}, kt)\times$

$$T(M(px_{2n+2}, qx_{2n+1}, qx_{2n+1}, kt), M(fgx_{2n+2}, px_{2n+2}, px_{2n+2}, kt))$$
$$\times M(hrx_{2n+1}, qx_{2n+1}, qx_{2n+1}, kt)$$

$$\geq (a(t)M(fgx_{2n+2}, px_{2n+2}, px_{2n+2}, t) + b(t)M(fgx_{2n}, hrx_{2n+1}, hrx_{2n+1}, t))$$
$$\times M(fgx_{2n+2}, qx_{2n+1}, qx_{2n+1}, 2kt).$$

Thus

$$M(y_{2n+2}, y_{2n+1}, y_{2n+1}, kt)\times$$

$$T(M(y_{2n+2}, y_{2n+1}, y_{2n+1}, kt), M(y_{2n+1}, y_{2n+2}, y_{2n+2}, kt))$$

$$\times M(y_{2n}, y_{2n+1}, y_{2n+1}, kt)$$

$$\geq (a(t)M(y_{2n+1}, y_{2n+2}, y_{2n+2}, t) + b(t)M(y_{2n+1}, y_{2n}, y_{2n}, t))$$
$$\times M(y_{2n+1}, y_{2n+1}, y_{2n+1}, 2kt).$$

Therefore

$$d_{2n+1}(kt) \geq d_{2n+1}(kt)\, T^{M(y_{2n+2}, y_{2n+1}, y_{2n+1}, kt),\, M(y_{2n}, y_{2n+1}, y_{2n+1}, kt)}$$
$$\geq a(t)d_{2n+1}(t) + b(t)d_{2n}(t)$$
$$\geq a(t)d_{2n+1}(kt) + b(t)d_{2n}(t).$$

Thus

$$(1 - a(t))d_{2n+1}(kt) \geq b(t)d_{2n}(t).$$

It follows that

$$d_{2n+1}(kt) \geq \frac{b(t)}{1 - a(t)} d_{2n}(t) = d_{2n}(t).$$

Hence for every $n \in$ ` we have $d_n(kt) \geq d_{n-1}(t)$. Now, we have

$$M(y_n, y_{n+1}, y_{n+1}, t) \geq M\left(y_{n-1}, y_n, y_n, \frac{t}{k}\right) \geq \ldots \geq M\left(y_0, y_1, y_1, \frac{t}{k^n}\right).$$

So, by Lemma 3.4, the sequence $\{y_n\}$ is Cauchy and by the completeness of X there is a y in X such that $\{y_n\}$ converges to y. Hence

$$\lim_{n\to\infty} px_{2n} = \lim_{n\to\infty} hrx_{2n} = \lim_{n} qx_{2n} = \lim_{n\to\infty} fgx_{2} = y.$$

+1 →∞ +1 n+2

Let $fg(X)$ be a closed subset of X, then there exists $v \in X$ such

that $fgv = y$. Putting $x = v$, $y = x_{2n+1}$ in (iii) we have

$M(pv, qx_{2n+1}, qx_{2n+1}, kt) \times$

$T(M(pv, qx_{2n+1}, qx_{2n+1}, kt), M(fgv, pv, pv, kt))$

$$\times M(hrx_{2n+1}, qx_{2n+1}, qx_{2n+1}, kt)$$

$$\geq (a(t)M(fgv, pv, pv, t) + b(t)M(fgv, hrx_{2n+1}, hrx_{2n+1}, t))$$

$$\times M(fgv, qx_{2n+1}, qx_{2n+1}, 2kt).$$

Letting $n \to \infty$, we get

$M(pv, y, y, kt) \times$

$(T(M(pv, y, y, kt), M(y, pv, pv, kt)) \times M(y, y, y, kt))$

$\geq (a(t)M(y, pv, pv, t) + b(t)M(y, y, y, t)) \times M(y, y, y, 2kt).$

Thus

$$M(pv, y, y, kt) \geq M(pv, y, y, kt) \times T(M(pv, y, y, kt), M(pv, y, y, kt))$$

$$\geq a(t)M(y, pv, pv, t) + b(t)$$

$$\geq a(t)M(y, y, pv, kt) + b(t).$$

So,

$$M(pv, y, y, kt) \geq \frac{b(t)}{1 - a(t)} = 1.$$

Hence $pv = y$. Since the pair (p, fg) is weakly compatible we have

$fgpv = pfgv$, hence $fgy = py$. Now from (iii), we have

$M(py, qx_{2n+1}, qx_{2n+1}, kt)\times$

$$T(M(py, qx_{2n+1}, qx_{2n+1}, kt), M(fgy, py, py, kt))$$

$$\times M(hrx_{2n+1}, qx_{2n+1}, qx_{2n+1}, kt)$$

$$\geq (a(t)M(fgy, py, py, t) + b(t)M(fgy, hrx_{2n+1}, hrx_{2n+1}, t))$$

$$\times M(fgy, qx_{2n+1}, qx_{2n+1}, 2kt).$$

Letting $n \rightarrow \infty$, we get

$$M(py, y, y, kt)\times$$

$$(T(M(py, y, y, kt), M(py, py, py, kt))\times M(y, y, y, kt))$$

$$\geq (a(t)M(y, py, py, t) + b(t)M(y, y, y, t))\times M(fgy, y, y, 2kt).$$

Thus

$$M(py, y, y, kt)\times M(py, y, y, 2kt)$$

$$\geq a(t)M(y, py, py, t) + b(t)\times M(py, y, y, 2kt)$$

It follows that

$$M(py, y, y, kt) \geq a(t)M(y, y, py, kt) + b(t),$$

so that,

$M(py, y, y, kt) \geq 1 - \frac{b(t)}{a(t)} = 1.$

Thus $py = y$. Hence $fgy = py = y$. Since $y = py \in p(X) \subseteq hr(X)$, there exists $w \in X$ such that $hrw = y$. From (iii), we have

$$M(py, qw, qw, kt)\times$$

$$T(M(py, qw, qw, kt), M(fgy, py, py, kt))$$

$$\times M(hrw, qw, qw, kt)$$

$$\geq (a(t)M(fgy, py, py, t) + b(t)M(fgy, hrw, hrw, t))$$

$$\times M(fgy, qw, qw, 2kt).$$

$$M(y, qw, qw, kt)\times$$

$$(T(M(y, qw, qw, kt), M(y, y, y, kt)) \times M(y, qw, qw, kt))$$

$$\geq (a(t)M(y, y, y, t) + b(t)M(y, y, y, t)) \times M(y, qw, qw, 2kt).$$

Thus

$$M(y, qw, qw, kt) \times M(y, qw, qw, kt)$$

$$\geq (a(t) + b(t))M(y, qw, qw, 2kt)$$

$$= M(y, qw, qw, 2kt).$$

Hence $M(y, qw, qw, kt) = 1$ so that $qw = y$.

Since the pair (q, hr) is weakly compatible, we have $hrqw = qhrw$ and

hence $hry = qy$. By (iii), we get

$$M(py, qy, qy, kt) \times T(M(py, qy, qy, kt), M(fgy, py, py, kt)) \times M(hry, qy, qy, kt)$$

$$\geq (a(t)M(fgy, py, py, t) + b(t)M(fgy, hry, hry, t)) \times M(fgy, qy, qy, 2kt).$$

Thus

$$M^2(y, qy, qy, kt)$$

$$\geq M(y, qy, qy, kt) \times T(M(y, qy, qy, kt), M(y, y, y, kt)) \times M(qy, qy, qy, kt)$$

$$= (a(t)M(y, y, y, t) + b(t)M(y, qy, y, kt)) \times M(y, qy, y, 2kt)$$

$$\geq a(t) + b(t)M(y, qy, y, kt) \times M(y, qy, y, kt).$$

This implies that

$$M(y, qy, y, kt) \geq \frac{a(t)}{1 - b(t)} = 1.$$

Hence $qy = y$. Since $fg = gf$ and $pg = gp$, we have

$fg(gy) = g(fgy) = gy$, and $pgy = gpy = gy$. Similarly, since $hr = rh$ and

$qr = rq$ we have $hr(ry) = r(hry) = ry$ and $qry = rqy = ry$. By (iii), we have

$M(pgy, qy, qy, kt) \times$

$T(M(pgy, qy, qy, kt), M(fg(gy), pgy, pgy, kt))$

$\times M(hry, qy, qy, kt)$

$\geq (a(t)M(fg(gy), pgy, pgy, t) + b(t)M(fg(gy), hry, hry, t))$

$\times M(fg(gy), qy, qy, 2kt)$.

Thus

$M(gy, y, y, kt) \times T(M(gy, y, y, kt), M(gy, gy, gy, kt))$

$\times M(y, y, y, kt)$

$\geq (a(t)M(gy, gy, gy, t) + b(t)M(gy, y, y, t)) \times M(gy, y, y, 2kt)$

Hence

$$M^2(gy, y, y, kt) \geq a(t) + b(t)M(gy, y, y, kt) \times M(gy, y, y, kt)$$

$$M(gy, y, y, kt) \geq a(t) + b(t)M(gy, y, y, kt)$$

$$M(gy, y, y, kt) \geq \frac{a(t)}{1 - b(t)} = 1.$$

It follows that $gy = y$. From (iii), we have

$M(py, qry, qry, kt) \times$

$T(M(py, qry, qry, kt), M(fgy, py, py, kt))$

$\times M(hry, qry, qry, kt)$

$\geq (a(t)M(fgy, py, py, t) + b(t)M(fgy, hry, hry, t))$

$\times M(fgy, qry, qry, 2kt)$.

Thus

$M^2(y, ry, ry, kt)$

$\geq M(y, ry, ry, kt) \times T(M(y, ry, ry, kt), M(y, y, y, kt))$

$$\times M(ry, ry, ry, kt)$$

$\geq (a(t)M(y, y, y, t) + b(t)M(y, ry, ry, t)) \times M(y, ry, ry, 2kt)$

$a(t) + b(t)M(y, ry, ry, kt) \times M(y, ry, ry, kt).$

Hence

$$M(y, ry, ry, kt) \geq \frac{a(t)}{1 - b(t)} = 1$$

so that $ry = y$. Therefore,

$$ry = gy = py = qy = fgy = hry = fy = hy = y.$$

Thus y is a common fixed point of the self maps of $f, g, h,$ $p,$ q and r.

To prove uniqueness, let x be another common fixed point of $f, g, h,$ $p,$ q and r.

Then

$M(px, qy, qy, kt) \times T(M(px, qy, qy, kt), M(fgx, px, px, kt))$

$$\times M(hry, qy, qy, kt)$$

$\geq (a(t)M(fgx, px, px, t) + b(t)M(fgx, hry, y, t)) \times M(fgx, qy, qy, 2kt).$

Thus

$M(x, y, y, kt) \times M(x, y, y, kt)$

$$\geq a(t) + b(t)M(x, y, y, t)\, M(x, y, y, 2kt)$$

$$\geq a(t) + b(t)M(x, y, y, kt)\, M(x, y, y, kt)$$

Therefore,

$$M(x, y, y, kt) \geq a(t) + b(t)M(x, y, y, kt).$$

Hence

$M(x, y, y, kt) \geq \frac{1}{1 - \frac{a(t)}{b(t)}} = 1.$

So $x = y$. □

Now we give an Example to illustrate our Theorem.

3.6 Example

Let $X = [0, 1]$, $T(a, b) = \min\{a, b\}$ and define mappings $f, g, h,$

$p, q, r : X \to X$ as

$$px = qx = gx = rx = 1,\ fx = hx = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

for all $x \in X$.

Let $a(t)$ and $b(t)$ be any arbitrary functions mapping from

$\mathbb{R}^+ \to (0, 1]$ such that $a(t) + b(t) = 1$ and

$$M(x, y, z, t) \geq \frac{t}{t + |x - y| + |y - z| + |z - x|}.$$

Then all conditions of Theorem 3.5 are satisfied and 1 is the unique common fixed point of f, g, h, p, q and r.

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