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A Numerical Study on Unsteady Natural Convection Flow with Temperature Dependent Viscosity past an Isothermal Vertical Cylinder

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Abstract: Aim of this paper is to investigate the boundary layer flow and heat transfer of unsteady laminar free convection flow past a semi-infinite isothermal vertical cylinder immersed in air. The fluid viscosity is adopted to vary the temperature. An explicit finite difference method has been devoted to solve the governing non-dimensional boundary layer equations. A parametric study is accomplished to interpret the influence of variable viscosity on the velocity and temperature profiles. The numerical consequences disclose that the viscosity has significant influence on transient velocity and temperature profiles, average skin friction coefficient and average heat transfer rate. The conclusion indicates that when the viscosity parameter increases the temperature and skin friction coefficient increases but the velocity near the wall and Nusselt number decreases. We have also shown the effect of viscosity variation parameter on isotherms and streamlines.

Keywords— Heat transfer, Natural convection, Variable viscosity, Vertical cylinder, Explicit finite difference method.

I. INTRODUCTION

Natural convection heat transfer has always been of particular interest among heat transfer problems. In natural convection, fluid motion is caused by natural means such as buoyancy due to density variations resulting from temperature distribution. Natural convection plays vital role in heat transfer in case of many applications such as electrical components transmission lines, heat exchangers and many other places. Many experiments have been performed during the last few decades and interesting results have been presented. Y.A. Cengel [1] discussed the natural convection phenomenon in case of vertical cylinder and governing equations to determine heat transfer coefficient. L. Davidson et al. [2] developed the natural convection phenomenon in vertical shell and tube. Also it was shown that for larger inlet velocity, there is a large value of Nusselt number. L. J. Crane [3] studied the natural convection over the vertical cylinder for very large Prandtl number and discussed the effect of high Prandtl number on convection through vertical cylinder. The effect of curvature of the cylinder where the thickness of the boundary layer is considerable was studied by C. O. Popie [4].

In all of the above studies, the authors assumed that the viscosities of the fluids are constant throughout the flow regime. For example, the viscosity of air is 0.6924 × 10-5 kg/m.s, 1.3289 kg/m.s, 2.286 kg/m.s. and 3.625 kg/m.s at 100K, 200K, 400K, and 800K temperature respectively Cebeci and Bradshaw [5]. In order to predict accurately the flow behavior, it is necessary to take into account the temperature dependence of viscosity Gary et al. [6] and Metha and Sood [7] found that the flow characteristics change substantially when the effect of temperature dependent viscosity are considered. The mixed convection boundary layer flow on a continuous flat plate with variable viscosity have also investigated by Hady et al. [8]. Kafoussias et al. [9] have studied the effects of variable viscosity on the free and mixed convection flow from a vertical flat plat in the region near the leading edge. Numerically unsteady natural convection of air and the effect of variable viscosity over an isothermal vertical cylinder was developed by H. P. Rani et al. [10] and concluded that as the viscosity increases the temperature and the skin friction coefficient increases while the velocity near the wall and Nusselt number decreases.

Actually less attention has been paid to the unsteady natural convection flow of a viscous incompressible fluid with variable viscosity over a heated vertical cylinder. The aim of the present work is to investigate the viscosity effects on the free convective flow of air past a semi-infinite vertical cylinder. The governing equations are solved numerically by explicit finite difference method to obtain the transient velocity, temperature, coefficient of skin friction, heat transfer rate, isotherms and streamlines for different values of the viscosity parameter.

II. MATHEMATICAL ANALYSIS OF THE PROBLEM

Consider an unsteady two dimensional natural convection boundary layer flow of a viscous incompressible fluid past an isothermal semi-infinite vertical cylinder of radius r_0.Here x is taken vertically upward along the axis of the cylinder and the origin of axis is taken to be at the leading edge of the cylinder where the boundary layer thickness is zero. It is assumed that the radial coordinate is perpendicular to the axis of the cylinder. Also the surrounding stationary fluid temperature is measured as the ambient temperature T^*_∞. Initially it is assumed that at time $t^*= 0$ the cylinder and the fluid are of the same temperature T^*_∞. When $t^*> 0$, the temperature of the cylinder is raised to T^*_w which is greater than the ambient temperature T^*_∞ and it gives rise to a buoyancy force. The effect of the viscous dissipation is measured negligible in the energy equation.

Under these assumptions the governing boundary layer equations for continuity, momentum and energy for the free convection flow over a vertical cylinder with Boussinesq's approximation are as follows

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial r}=0 \qquad (1)$$

$$\frac{\partial u}{\partial t^*}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=g\beta\left(T^*-T^*_\infty\right)+\frac{1}{\rho r}\frac{\partial}{\partial r}\left(\mu r\frac{\partial u}{\partial r}\right) \qquad (2)$$

$$\frac{\partial T^*}{\partial t^*}+u\frac{\partial T^*}{\partial x}+v\frac{\partial T^*}{\partial r}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T^*}{\partial r}\right) \qquad (3)$$

with initial and boundary conditions

$$\begin{aligned}
&t^*\le 0: u=0, v=0, T^*=T^*_\infty && \text{for all } x \text{ and } r\\
&t^*>0: u=0, v=0, T^*=T^*_\infty && \text{at } r=r_0\\
&u=0, v=0, T^*=T^*_\infty && \text{at } x=0 \qquad (4)\\
&u\to 0, v\to 0, T^*\to T^*_\infty && \text{as } r\to\infty
\end{aligned}$$

To get the solution of the equation (1) to (3) along with (4) we want to make them nondimensional. For this purpose we use the following nondimensional quantities

$$X=Gr^{-1}\frac{x}{r_0},\ R=\frac{r}{r_0},\ U=Gr^{-1}\frac{ur}{\nu},\ V=\frac{vr_0}{\nu},\ t=\frac{\nu t^*}{r_0^2}$$
$$T=\frac{T^*-T^*_\infty}{T^*_w-T^*_\infty},\ Gr=\frac{g\beta r_0^3\left(T^*_w-T^*_\infty\right)}{\nu^2},\ \Pr=\frac{\nu}{\alpha} \qquad (5)$$

We assume viscosity of fluid to be proportional to a linear function of temperature as

$$\mu(T)=\mu_\infty(1+\gamma T) \qquad (6)$$

where μ_∞ is the viscosity of ambient fluid, T is the dimensionless temperature and γ is a scalar parameter which shows the influence of temperature on variable viscosity.

By introducing the non dimensional variables of (5) and using (6) into the equations (1) to (3) along with (4), we get the following nondimensional equations (7) to (9)

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial R}+\frac{V}{R}=0 \qquad (7)$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial R}=T+(1+\gamma T)\left(\frac{\partial^2 U}{\partial R^2}+\frac{1}{R}\frac{\partial U}{\partial R}\right)+\gamma\frac{\partial T}{\partial R}\frac{\partial U}{\partial R} \qquad (8)$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial R}=\frac{1}{\Pr}\left(\frac{\partial^2 T}{\partial R^2}+\frac{1}{R}\frac{\partial T}{\partial R}\right) \qquad (9)$$

The corresponding initial and boundary conditions in non-dimensional variables are reduced to the following form

$$\begin{aligned} &t \le 0: U = 0, V = 0, T = 0 \quad \text{for all } X \text{ and } R \\ &t > 0: U = 0, V = 0, T = 1 \quad \text{at } R = 1 \\ &U = 0, V = 0, T = 0 \quad \text{at } X = 0 \\ &U \to 0, V \to 0, T \to 0 \quad \text{as } R \to \infty \end{aligned} \tag{10}$$

III. NUMERICAL ANALYSIS OF THE PROBLEMA

In order solve the non linear governing equations (7)-(9) along with (10) an explicit finite difference method has been employed. The finite difference equation corresponding to equations (7)-(9) get the equations (11) to (13) respectively

$$\frac{U(i,j)-U(i-1,j)}{\Delta X}+\frac{V(i,j)-V(i-1,j)}{\Delta R}+\frac{V(i,j)}{1+(j-1)\Delta R}=0 \tag{11}$$

$$\begin{aligned} &\frac{U'(i,j)-U(i,j)}{\Delta\tau}+U(i,j)\frac{U(i,j)-U(i-1,j)}{\Delta X} \\ &+V(i,j)\frac{U(i,j+1)-U(i,j)}{\Delta R}=T(i,j)+\left[1+\gamma T(i,j)\right] \\ &\left[\frac{U(i,j+1)-2U(i,j)+U(i,j-1)}{(\Delta R)^2}+\frac{1}{\left[1+(j-1)\Delta R\right]}\frac{U(i,j+1)-U(i,j)}{\Delta R}\right] \\ &+\gamma\frac{T(i,j+1)-T(i,j)}{\Delta R}\frac{U(i,j+1)-U(i,j)}{\Delta R} \end{aligned} \tag{12}$$

$$\begin{aligned} &\frac{T'(i,j)-T(i,j)}{\Delta\tau}+U(i,j)\frac{T(i,j)-T(i-1,j)}{\Delta X}+V(i,j)\frac{T(i,j)-T(i-1,j)}{\Delta R} \\ &=\frac{1}{\Pr}\left[\frac{T(i,j+1)-2T(i,j)+T(i,j-1)}{(\Delta R)^2}+\frac{1}{\left[1+(j-1)\Delta R\right]}\frac{T(i,j+1)-T(i,j)}{\Delta R}\right] \end{aligned} \tag{13}$$

To obtain the finite difference equations the region of the flow is divided into the grids or meshes of lines parallel to X and R is taken normal to the axis of the cylinder. Here we consider that the height of the cylinder is X_{max}=100 i.e. X varies from 0 to 100 and regard R_{max}=25 as corresponding to $R\rightarrow\infty$ i.e. R varies from 0 to 25. In the above equations (11) to (13) the subscripts i and j designate the grid points along the X and R coordinates, respectively, where X = iΔX and R = 1+(j-1)ΔR .There are m=500 and n=500 grid spacing in the X and R directions respectively.

From the initial and boundary conditions given in equation (10), the values of velocity U,V and temperature T are known at time τ = 0; then the values of U,V and T at the next time step can be evaluated. Generally, when the above variables are known at τ = n$\Delta\tau$, the values of variables at τ = (n+1)$\Delta\tau$ are calculated as follows. The finite difference equations (12) and (13) at every internal nodal point on a particular i -level constitute a tri-diagonal system of equations. Such a system of equation is solved by Thomas algorithm. At first the temperature T is calculated from equation (12) at every j nodal point on a particular i -level at the (n+1) time step. By making the use of these known values of T, the velocity U at the (n+1) time step is calculated from equation (11) in a similar way. Thus the values of T and U are known at a particular i -level. Then the velocity V is calculated from equation (10) explicitly. This process is repeated for the consecutive i -levels. Thus the values of U, V and T are known at all grid points in the rectangular region at the (n+1)th time step. This iterative procedure is repeated for many time steps until the steady state solution is reached.

IV. RESSULTS AND DISCUSSION

We have obtained numerical solutions by solving the finite difference equations using explicit finite difference method. The velocity, temperature, coefficient of skin friction, rate of heat transfer in terms of Nusselt number, isotherms and streamlines have been carried out by assigning some arbitrarily chosen specific values to the physical parameters involved in the problem. Also for each feasible difference of wall and ambient temperature it can be said that the variation of the Prandtl number with temperature is not noticeable. Therefore, the non dimensionalized system of equations (7)-(9) along with (10) have been solved with a fixed value of Prandtl number. In the present numerical solution four values of γ are chosen 0.2, 0.4, 0.6, and 0.8 with a fixed value of Prandtl number Pr = 0.70. In case of isotherms and streamlines we have used another four values of γ 0.25, 0.50, 0.75 and 1.00. The figures computed from the numerical method of the problem have been displayed in Figs. (1-9).
The present velocity and temperature profiles are compared with the results of H. P. Rani et al. [10] for the steady state, isothermal and constant thermal conductivity with Pr = 0.7. The comparison results, which are shown in Fig. 1and Fig. 2 are found to be in good agreement.

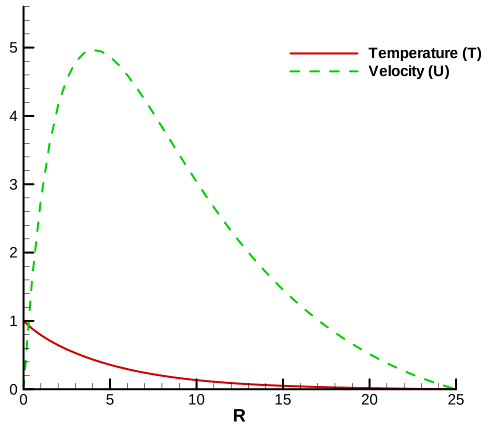


Fig.1. Comparison of the velocity and temperature profiles for Pr = 0.7 and γ = 0.2

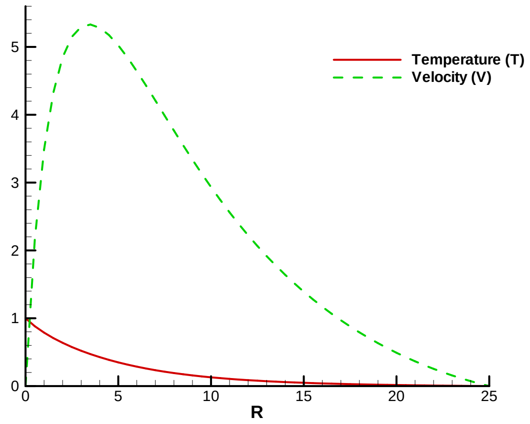


Fig.2. Comparison of the velocity and temperature profiles for Pr = 0.7 and γ = - 0.2.

VELOCITY

Fig. 3 and Fig. 4 illustrate the graphical representation of the simulated transient velocity profiles at the temporal maximum and steady state against the radial coordinate *R* at *X*=1.0 for different γ. It is noticed that the velocity profiles start with the value zero at the wall, reached their maximum close to the hot wall and then monotonically decrease to zero. It is clear that the time to reach the temporal maximum of velocity increases with the increasing viscosity variation parameter γ, while the time to reach the steady- State are almost the same for different γ. It is noticed that the magnitude of the peak velocity becomes smaller as viscosity variation parameter becomes larger. It is observed that if we increase values of the viscosity variation parameter then it increases the velocity of the flow away from the wall, because the viscosity is increasing with the increase of the viscosity variation parameter. The location of the maximum velocity gets far away from the cylinder for higher values of γ. This qualitative arises because, for the case of fluid with larger viscosity (say γ = 0.8), the fluid is not capable to move easily in a region very near the heated surface, while the fluid with smaller viscosity (say γ = 0.2) can move more freely close to the wall. From the above discussion, it is clear that neglecting the variation of the fluid viscosity, which depends on the temperature, introduces a substantial error.

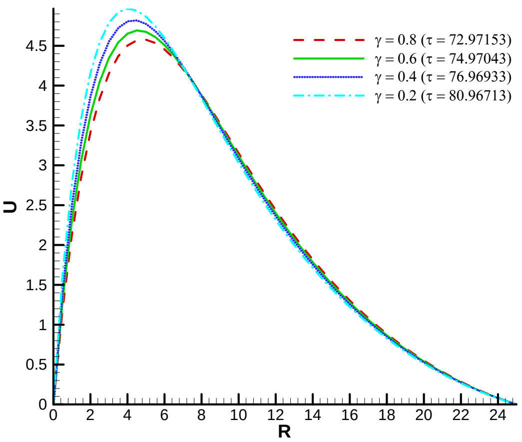


Fig.3.Variation of the steady state velocity profiles with respect to positive values of γ.

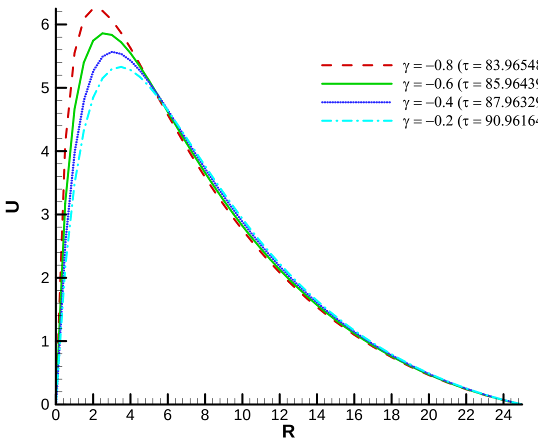


Fig.4.Variation of the steady state velocity profiles with respect to negative values of γ.

TEMPERATURE

Fig. 5 and Fig. 6 show the graphical representation of the simulated steady state temperature profiles against the radial coordinate *R* at *X*=1.0 for different γ. It is observed that the temperature profiles start with the hot wall temperature (*T*=1) and then monotonically decrease to zero as the radial coordinate increases. Also it is noticed that temperature profiles increase with the increase of the viscosity variation parameter. It is connected to the matter that with the increase in the viscosity variation parameter the viscosity of the fluid is increases, which permits higher velocity away from the hot wall.

The temperature profiles increase with increasing γ, which is related with the fact that the increase in γ causes the decrease in the peak velocity as shown in Fig. 3.and Fig. 4 However, two opposite effects of the increase in γ on the fluid particle can be considered.

The first effect decreases the velocity of the fluid due to increase in viscosity where the second effects increase the velocity of the fluid particle due to increase in temperature as shown in Fig. 5 and Fig. 6. Close to the cylinder wall the temperature is relatively high, as a results, the first effect will be dominant and the velocity decreases as γ increases (Fig. 3 and Fig. 4). On the other hand, away from the cylinder wall, where the temperature T is relatively low, the second effect will be dominant and the velocity increases as γ increases (Fig. 3 and Fig. 4).

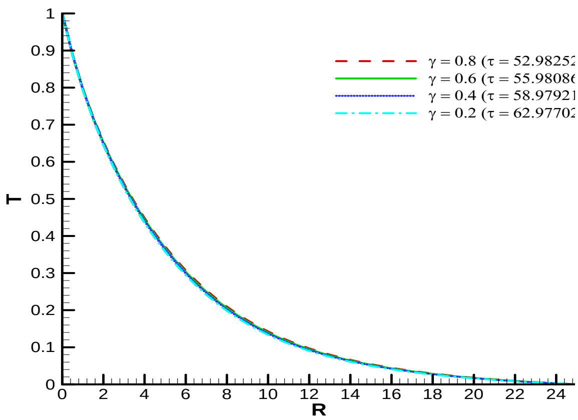


Fig.5.Variation of the steady state temperatures profiles with respect to positive values of γ.

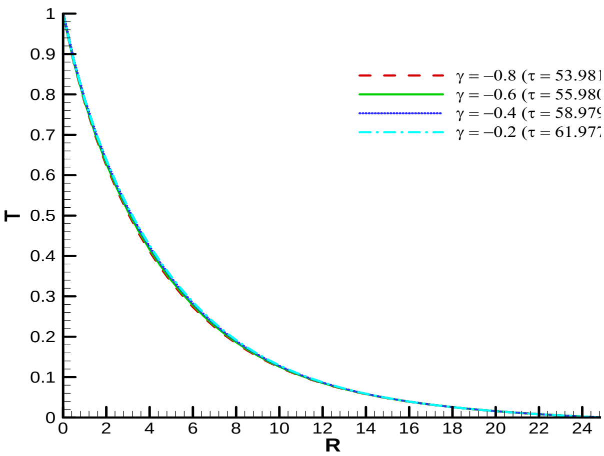


Fig.6.Variation of the steady state temperatures profiles with respect to negative values of γ.

AVERAGE SKIN FRICTION COEFFICIENT AND HEAT TRANSFER RATE

We have calculated average skin friction coefficient as

$$\overline{C_f} = (1+\lambda)\int_0^1 \left(\frac{\partial U}{\partial R}\right)_{R=1} dX \qquad (16)$$

The average heat transfer rate (Nusselt number) is expressed as

$$\overline{Nu} = -\int_0^1 \left(\frac{\partial T}{\partial R}\right)_{R=1} dX \qquad (17)$$

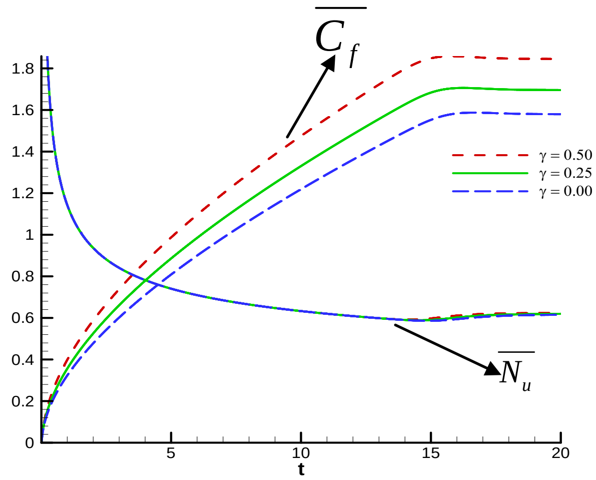
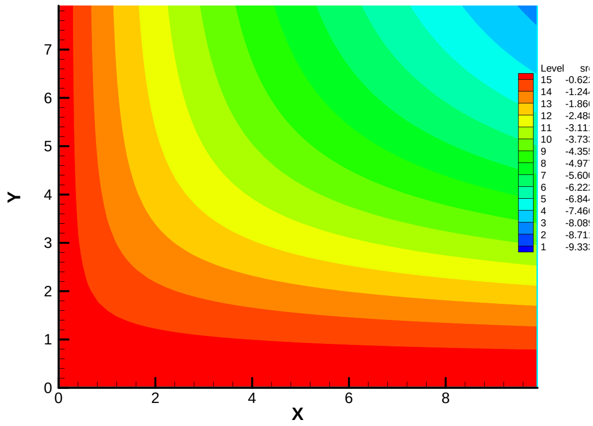


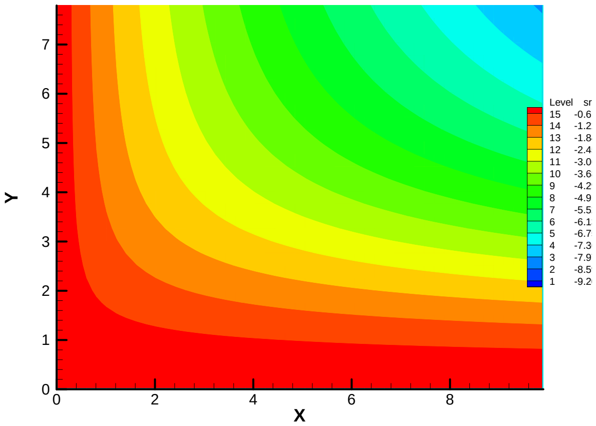
Fig. 7. Variation of the average skin friction and Nusselt number with respect to γ

In Fig.7, the simulated average non dimensional skin friction coefficient and heat transfer rate for different γ have been plotted against the time. It is noticed from the figure that for all values of γ the average skin friction coefficient increases with time, reaches the temporal maximum value and after little decreasing, becomes asymptotically steady. For increasing value of γ the average skin friction increases in association with the fact that the increase in value of γ yields the increase in the viscosity near the wall. The Nusselt number for different values of γ decrease with time at the beginning, reach the temporal minimum and then after slide increasing, reach the steady state. For different values of γ there is slide difference in the average Nusselt number in the very early part of the transient period. This fact explains that initially the heat transfer is performed mostly by the conduction with the large temperature difference between the wall and the fluid. With the increase of time, the free convection effect becomes more pronounced and as a result, the local Nusselt number generally decreases, decreasing the heat transfer rates.

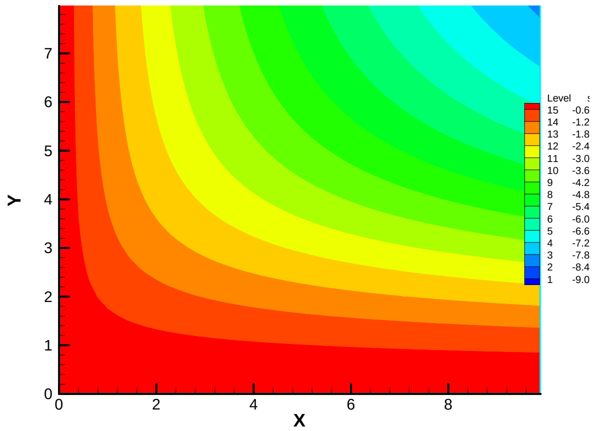
STREAMLINES AND ISOTHERMS



(a)

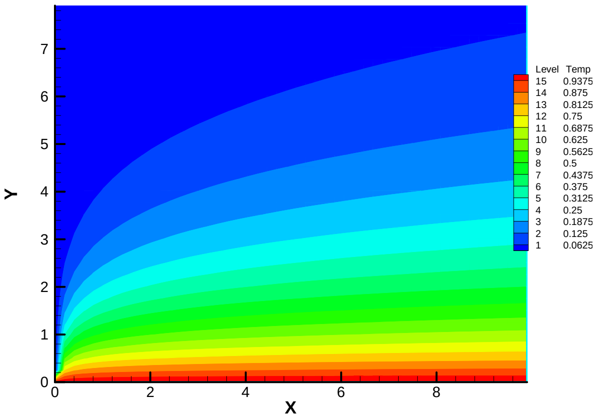


(b)

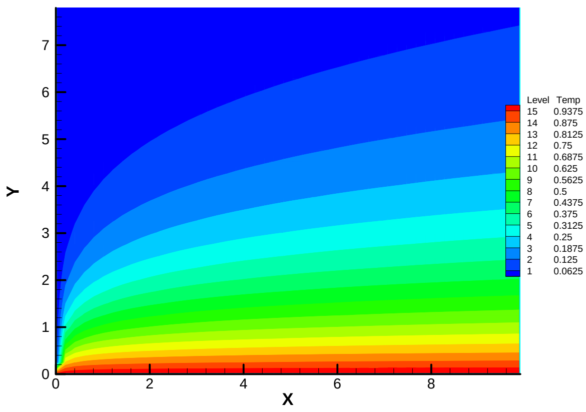


(c)

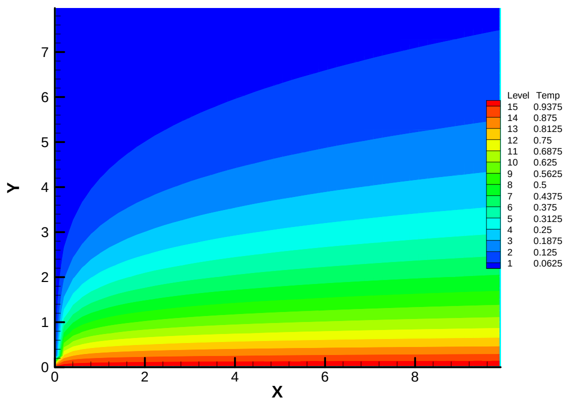
Fig. 8. (a), (b), (c) represent the streamlines with respect to γ=0.00 , γ=0.25 and γ=0.50 respectively.



(a)



(b)



(c)

Fig. 9. (a), (b), (c) represent the isotherm lines with respect to $\gamma=0.00$, $\gamma=0.25$ and $\gamma=0.50$ respectively.

Figure 8 is showing the streamline for different values of the viscosity variation parameter γ. It is observed from the figure that without the effect of viscosity variation parameter (i.e. γ = 0.0), the values of stream are lower when the boundary layer thickness is highest shown in figure 8(a), but with the increase of viscosity variation parameter γ (when γ = 0.25 and γ = 0.50) increases the values of stream shown in figure 8(b) and 8(c), also the momentum and boundary layer become thinner. Figure 8 illustrates the effect of the viscosity variation parameter γ on the development of isotherms which is plotted for γ (= 0.00, 0.25, and 0.50). From the figure it is clearly noticed that the viscosity of the fluid increased at the vicinity of the surface which indicates that the viscosity of the fluid is strongly dependent on temperature. The temperature distribution reduces slightly for large values of γ. Finally it can be concluded saying that the momentum and thermal boundary layer become thin for high viscose fluid.

V. CONCLUTION

Numerical study for the unsteady natural convection of air with variable viscosity along a semi-infinite vertical cylinder has been investigated. The viscosity of the fluid is assumed to be temperature dependent, while the Prandtl number is kept constant. An explicit method is used to solve the dimensionless governing equations in a meridian plane. The computations are carried out to study the influence of the viscosity variation parameter γ on the transient dimensionless velocity, temperature, skin friction coefficient and heat transfer rate, streamlines and isotherms.

Generally less attention has been paid to the unsteady natural convection flow of a viscous incompressible fluid with variable viscosity over a heated vertical cylinder. The aim of the present work is to investigate the viscosity effects on the free convective flow of air past a semi-infinite vertical cylinder. From the present numerical analysis the following observations are established.

Velocity profiles near the wall decrease with the increase of γ, while the temperature profiles increase. The time which is taken to reach the temporal maximum of the velocity increases with the increase of γ. Initially, the unsteady behavior of the temperature with the variable viscosity coincides with that of fluid with constant properties. Then the temperature with the variable viscosity deviates from that with constant properties and reached the steady state asymptotically. When the viscosity variation parameter is larger, lower velocity near the isothermal cylinder wall and higher velocity in a region away from the wall are observed, which gives the lower average Nusselt number. The increase in the viscosity variation parameter leads to the decrease in the average heat transfer rate and to the increase in the average skin friction. With the increase of viscosity variation parameter γ increases the values of stream and the temperature distribution reduces slightly for large values of γ. As a result the momentum and thermal boundary layer become thin for high viscose fluid.

The results of our study are compared with the results of H. P. Rani et al. [10] for the steady state, isothermal and constant viscosity with Pr = 0.7. The comparison results are found to be in good agreement. Additionally we have also showed the effect of viscosity variation parameter γ on streamlines and isotherms.

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