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Testing the weak form of efficient market hypothesis: evidence from the Moroccan Stock Market

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Abstract: Objective: The aim of this study is to evaluate the weak form informational efficiency of the Moroccan stock market through its MASI index (Moroccan All Shares Index).

Methodology: To this end, we used the daily closing prices of the MASI index, covering the period from 03/01/2002 to 23/11/2024, totaling 5302 observations. To assess the random walk hypothesis (RWH), various tests were applied, namely unit root tests, including the Augmented Dickey-Fuller test, the Phillips-Perron test, and the Kwiatkowski, Phillips, Schmidt, and Shin test; normality tests such as the Jarque-Bera test, the Kolmogorov-Smirnov test, the Lilliefors test, the Cramer-von Mises test, the Anderson-Darling test, and the quantile-quantile plot-based test; the Ljung-Box serial correlation test; the variance ratio test; and the Runs test.

Results: The results obtained from all applied tests indicate that the series of geometric returns of the MASI index does not follow a random walk.

Conclusion: Based on the results of the various tests applied, it is concluded that the Moroccan stock market is inefficient in its weak form.

Keywords: Weak form informational efficiency, random walk hypothesis, unit root test, normality tests, variance ratio test, runs test.

JEL Classification: C12, C13, C14, C22, G11, G14,

1. Introduction

Formulated in the 1960s by economist Eugene Fama, the Efficient Market Hypothesis (EMH) posits that financial markets tend to be efficient. This theory suggests that the prices of financial assets incorporate all available information, making it challenging to achieve exceptional profits through fundamental or technical analysis. Numerous studies have been conducted with the aim of providing empirical evidence supporting the Efficient Market Hypothesis (EMH).

Fama (1970) categorized efficient markets into three forms, namely weak-form efficiency, semi-strong-form efficiency, and strong-form efficiency:

1) Weak-form efficiency: Current asset prices reflect all past information, meaning that technical analysis based on price history should not enable the prediction of future price movements.
2) Semi-strong-form efficiency: Current asset prices reflect all public information, including both past and current information, as well as all publicly available information. This implies that neither fundamental nor technical analysis should lead to exceptional profits.
3) Strong-form efficiency: Current asset prices reflect all information, whether public or private. According to this form of efficiency, even access to privileged information should not lead to abnormal profits.

Despite the extensive studies and debates surrounding the Efficient Market Hypothesis, it is not immune to criticism. Some dissenters argue that market efficiency is not constant due to irrational investor behavior, information asymmetry, transaction costs, and other factors. Financial bubbles and historical stock market crashes are often cited as examples to challenge the idea of perfect market efficiency.

Ultimately, the Efficient Market Hypothesis remains a crucial theoretical framework for understanding the workings of financial markets, although its status sparks debates and controversies within the financial domain.

This study aims to assess the weak form efficiency hypothesis of the Moroccan stock market. Its significance lies in the fact that the conclusions obtained will provide new evidence to the existing literature on frontier and emerging stock markets, thereby strengthening the available empirical knowledge. Furthermore, this research is expected to benefit both regulators and investors/traders.

To evaluate the weak form efficiency hypothesis, various statistical tests, both parametric and non-parametric, were employed in this study. These tests include unit root tests, normality tests, Ljung-Box serial correlation test, variance ratio test, and runs test. The purpose of these tests is to examine the random walk hypothesis (RWH).

The remainder of this study is organized as follows. Section two reviews the literature. Section three describes the data and methodology used in the study. Section four presents and discusses the results obtained. Section five concludes.

2. Literature Review

A significant number of studies worldwide have been conducted with the aim of seeking empirical evidence supporting the Efficient Market Hypothesis (EMH). Among these studies, the one conducted by Fama in 1965 holds a prominent place in the literature. Widely cited, this study applied the Runs test, Alexander's filter technique, and the serial correlation test on the daily returns of 30 individual stocks listed on the Dow Jones Industry market, covering the period from 1957 to 1962. Fama's results revealed an insignificant correlation, leading to the conclusion that, the average Dow Jones Industry market was weak-form efficient.

Another study, conducted by Smith and Ryoo (2003), aimed to test the efficiency hypothesis for five emerging European markets, namely Greece, Hungary, Poland, Portugal, and Turkey, using the multiple variance ratio test. In four of the markets, the random walk hypothesis was rejected due to the autocorrelation of returns. For the Istanbul market, characterized by significantly higher trading volume than the other markets in the 1990s, the stock price index follows a random walk.

Nisar and Hanif (2012) examined the weak form of the market efficiency hypothesis on the four major stock exchanges in South Asia, namely India, Pakistan, Bangladesh, and Sri Lanka. They applied four statistical tests, including the Runs test, serial correlation, unit root test, and variance ratio test, to monthly, weekly, and daily returns over a period of 14 years (1997-2011). The results suggest that none of the four major stock exchanges in South Asia follows a random walk, indicating the absence of weak form efficiency in these markets.

Kumar and Singh (2013) conducted a study to assess the weak form efficiency of the Indian stock market. They used the daily closing values of the S&P CNX Nifty and CNX Nifty Junior for the period from January 1, 2000, to March 31, 2013. Tests such as the Kolmogorov-Smirnov normality test, Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root tests, as well as the Runs test, revealed that the Indian stock markets do not exhibit weak form market efficiency.

Shaker (2013) examined the weak form efficiency of the Finnish and Swedish stock markets using tests such as the serial correlation test, Augmented Dickey-Fuller unit root test, and variance ratio test. These tests were applied to the daily returns of the OMX Helsinki and OMX Stockholm indices from 2003 to 2012. The results showed that daily returns do not adhere to the assumption of a normal distribution, thus concluding that the Finnish and Swedish stock markets are not weak-form efficient.

Chiny and Mir (2015) studied the informational efficiency of the Moroccan stock market through four indices (Masi, BNQ, ASSUR, and IMMO), using daily prices over the period from January 2002 to December 2013. The results of various tests formally rejected the weak form efficiency hypothesis for these markets.

Khaled (2017) examined the informational efficiency of the Algerian stock market by applying the variance ratio test to the daily prices of the DZAIRINDEX over the period from January 2008 to December 2016. The results indicated the inefficiency of the Algerian stock market in the weak form, thereby rejecting the presence of a random walk process in the index price series.

Kheertee et al. (2017) studied the Random Walk Hypothesis on four indices of the Mauritanian stock market SEM (SEMDEX, SEMTRI, DEMEX, and DEMT.RI) over the daily period from August 2006 to May 2014. The results showed that the returns of SEMDEX are stationary, while those of SEMTRI, DEMEX, and DEMTRI contain a unit root. The presence of a unit root implies that SEM is efficient, but referring to one of the main indices, SEMDEX, SEM is considered inefficient with possibilities for additional gains.

Elhami and Hefnaoui (2018) conducted an analysis of the daily and weekly returns of stock indices from emerging and frontier markets in the Middle East and North Africa (MENA) region over the period from January 2010 to August 2017. They selected four indices from emerging markets and four indices from frontier markets in the MENA region, applying tests such as autocorrelation, runs test, unit root tests, and multiple variance ratio test. The results are mixed depending on the indices, but overall, the return series indicate a lack of market efficiency. The general conclusion suggests that stock index returns do not follow a random walk in MENA region countries, thus opening the possibility for investors to achieve arbitrage profits due to market inefficiency.

Erdas (2019) studied the informational efficiency of the stock markets in Central and Eastern Europe (CEEC), including countries such as Lithuania, Hungary, Romania, Croatia, Slovenia, Poland, Bulgaria, Slovakia, Latvia, Estonia, and the Czech Republic. He applied linearity tests and unit root tests to both linear and non-linear series of the respective markets. The results indicate that most markets have linear characteristics, while others exhibit non-linear features. Unit root tests reveal that, over the analyzed period, the weak form market efficiency hypothesis is validated in the CEEC, suggesting that investors should not be able to achieve abnormal returns by analyzing historical prices in these countries.

Tas and Atac (2019) examined the weak form efficiency of the Istanbul Stock Exchange (ISE) using the Random Walk Hypothesis (RWH). The results from the Dickey-Fuller and runs tests are contrasting. While the Dickey-Fuller test suggests that weak form market efficiency is not justified at the Istanbul Stock Exchange, the runs test does not provide conclusive results on market efficiency.

Abouahmed (2019) studied the informational efficiency of the Moroccan stock market over the period from January 2013 to December 2015. Normality tests and unit root tests indicated that the series of monthly returns of the MADEX follows a normal distribution and is stationary. The empirical results confirmed the informational efficiency of the Moroccan stock market during this period.

Suadiq and Vural (2020) conducted an empirical study assessing the weak form efficiency of stocks in the banking sector of the Istanbul Stock Exchange. The results of autocorrelation tests, runs tests, and unit root tests showed variations in the efficiency of different samples. Some were identified as effective in terms of weak efficiency, while others were not, according to different methods.

Faiteh and Najab (2020) analyzed the link between the organizational structure of the Moroccan stock market and its efficiency. Their results suggested that the Moroccan stock market is inefficient in the weak form, attributing this inefficiency to structural shortcomings such as transaction costs, brokerage functions monopolized by the banking sector, and dependence on regulatory bodies.

Ifleh and El Kabbouri (2021) tested the random walk hypothesis in the Moroccan financial market over the period from January 2012 to January 2021. The results of unit root tests rejected the random walk hypothesis, raising questions about the weak form informational efficiency of the Moroccan stock market.

3. Data and Methodology

3.1 Data

The data used in this study consists of the daily closing prices of the Casablanca Stock Exchange index (MASI), covering the period from 03/01/2002 to 23/11/2024, totaling 5502 observations. The MASI (Moroccan All Shares index) is a capitalization-weighted index that includes all equity-type securities listed on the Casablanca Stock Exchange. It is thus a broad index, providing an optimal way to track the development of the entire population of listed securities.

The MASI index prices were then converted into geometric returns:

$$r_t = ln\left(\frac{P_t}{P_{t-1}}\right) = ln(P_t) - ln(P_{t-1}) \quad (1)$$

The data was downloaded from the website www.investing.com.

3.2 Methodology

In this section, we will explore various statistical tests related to the random walk hypothesis in the context of the Moroccan stock market. Firstly, we will address stationarity tests, including the Augmented Dickey-Fuller test (ADF), the Phillips-Perron test (PP), and the Kwiatkowski, Phillips, Schmidt, and Shin test (KPSS). Secondly, we will examine normality tests, such as the Jarque-Bera test, the Kolmogorov-Smirnov test, the Lilliefors test, the Cramer-von Mises test, the Anderson-Darling test, as well as the test based on quantile-quantile plots, often abbreviated as QQ plots. Thirdly, we will delve into the autocorrelation test, namely the Ljung-Box test. Fourthly, we will clarify the variance ratio test. Finally, we will proceed with the description of the runs test.

3.2.1 Unit Root Tests (Stationarity Tests)

Unit root tests are frequently employed in statistical analyses to study the random characteristic of return series. Essentially, these tests are conducted to explore the presence of a unit root, indicating the non-stationarity of return series.

While the presence of a unit root is not sufficient to confirm the random nature of a series, it remains a necessary condition for this random behavior. That's why many researchers resort to unit root tests to test the weak form efficiency hypothesis.

Three types of unit root tests are commonly used to examine the random behavior of the return series, namely the Augmented Dickey-Fuller test invented by Dickey and Fuller (1979), the Phillips-Perron (PP) test invented by Phillips and Perron (1988), and the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test invented by Kwiatkowski et al. (1992).

The Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests use the following null and alternative hypotheses:

H_0 : The series contains a unit root (non-stationary)

H_1 : The series does not contain a unit root (stationary)

The Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test uses the following null and alternative hypotheses:

H_0 : The series does not contain a unit root (stationary)

H_1 : The series contains a unit root (non-stationary)

- **Augmented Dickey-Fuller Test (ADF)**

The Augmented Dickey-Fuller (ADF) test used to assess the presence of unit roots in a time series. The specification of the ADF test can be formulated as follows:

1) Regression Model: The regression equation used in the ADF test is based on an autoregressive (AR) model of order p, where p is the number of lags to include. The general form of the model is as follows:

$$\Delta Y_t = \alpha + \beta t + \gamma Y_{t-1} + \sum_{i=1}^{p} \delta_i \, \Delta Y_{t-i} + \varepsilon_t \tag{2}$$

where

Δ represents the first difference operator defined by:

$$\Delta Y_t = Y_t - Y_{t-1}$$

Y_t represents the geometric return of the stock index on date t.

Y_{t-1} is the lagged value of the time series.

α is the constant term.

β is the coefficient of the linear trend.

t is the time variable.

γ is the coefficient of the lagged value of the series.

δ_i are the coefficients associated with the first differences of the series at lags i.

p is the order of the autoregressive process.

ε_t is the error term.

2) Null and Alternative Hypotheses: The null hypothesis of the ADF test is that the time series Y_t has a unit root, indicating non-stationarity. The alternative hypothesis is that the series does not have a unit root, indicating stationarity.

There are three main versions of the test:

Model 1: Unit root test (without constant and trend)

$$\Delta Y_t = \gamma Y_{t-1} + \sum_{i=1}^{p} \delta_i \, \Delta Y_{t-i} + \varepsilon_t \tag{3}$$

Model 2: Unit root test with constant

$$\Delta Y_t = \alpha + \gamma Y_{t-1} + \sum_{i=1}^{p} \delta_i \, \Delta Y_{t-i} + \varepsilon_t \tag{4}$$

Model 3: Unit root test with constant and time trend

$$\Delta Y_t = \alpha + \beta t + \gamma Y_{t-1} + \sum_{i=1}^{p} \delta_i \, \Delta Y_{t-i} + \varepsilon_t \qquad (5)$$

Each version of the test has its own critical value that depends on the sample size. In each case, the null hypothesis is that there is a unit root, $\gamma = 0$.

3) Parameters Estimation: The model parameters are estimated using the least squares method.

4) Calculation of the Test Statistic: Under the null hypothesis, the t-statistic for γ is given by:

$$ADF = t_\gamma = \frac{\hat{\gamma}}{se(\hat{\gamma})} \qquad (6)$$

where $\hat{\gamma}$ is the estimate of γ and $se(\hat{\gamma})$ is the standard deviation of $\hat{\gamma}$.

5) Choice of the number of lags p: The selection of the number of lags p in the model often depends on criteria such as the Bayesian Information Criterion (BIC) or the Akaike Information Criterion (AIC).

6) Comparison with critical values and decision-making: The ADF test statistic is compared to critical values tabulated or obtained through simulation to determine whether the null hypothesis can be rejected.

If the calculated absolute value of the t-statistic exceeds the critical values of the DF or MacKinnon t-statistic at the significance level α (1% or 5% or 10%), the null hypothesis $\gamma = 0$ is rejected, in which case the time series is stationary. If the calculated absolute value of the t-statistic does not exceed the critical values of t, the null hypothesis is not rejected at the significance level α =1% or 5% or 10%, in which case the time series is non-stationary.

➢ **Phillips-Perron (PP) Test**

The Phillips-Perron (PP) test, proposed by Phillips and Perron (1988), is a unit root test similar to the Dickey-Fuller test. It is used to assess the presence of a unit root in a time series, indicating non-stationarity. The Phillips-Perron test has a specification similar to that of the Dickey-Fuller test, but it has the advantage of being robust to serial correlation and heteroscedasticity issues in the residuals.

The specification of the Phillips-Perron test is as follows:

1) Regression Model: The Phillips-Perron test uses an augmented autoregressive (AR) regression model to model the time series. The model is defined as follows:

$$\Delta Y_t = \alpha + \beta t + \gamma Y_{t-1} + \sum_{i=1}^{p} \delta_i \, \Delta Y_{t-i} + \varepsilon_t \qquad (7)$$

where

Δ represents the first difference operator defined by:

$$\Delta Y_t = Y_t - Y_{t-1} \qquad (8)$$

Y_t represents the geometric return of the stock index on date t.

Y_{t-1} is the lagged value of the time series.

t is the time variable.

$\alpha, \beta, \gamma, \delta_i$ are the parameters to be estimated.

p is the order of the autoregressive process.

ε_t is the error term.

2) Correction of the deterministic trend: The Phillips-Perron test includes a correction for the deterministic trend to address trend issues in the time series. This correction can be linear or include quadratic or cubic trend terms.

3) Correction for autocorrelated and heteroscedastic errors: The Phillips-Perron test uses robust estimation of standard errors that takes into account autocorrelation and heteroscedasticity issues in the residuals.

4) Test statistic: The Phillips-Perron test statistic is based on the average of the t-statistics obtained for the unit root coefficients in the regression equation.

$$PP = \frac{T \times \bar{t}}{\sigma} \qquad (9)$$

where :

T is the sample size.

$\bar{t}$ is the mean of the t-statistics of the coefficients of the unit root.

$\sigma$is the standard deviation of these t-statistics.

5) Asymptotic Distribution: The asymptotic distribution of the test statistic depends on the presence or absence of serial correlation and other characteristics of the time series. Tables of critical values are used to determine if the test statistic is significant.
6) Decision Making: If the test statistic is below the critical threshold, the null hypothesis of the presence of a unit root is rejected, suggesting that the series is stationary. Otherwise, the null hypothesis is not rejected, indicating non-stationarity of the time series.

The Phillips-Perron (PP) test and the Augmented Dickey-Fuller (ADF) test are two unit root tests that share a similar objective, although there are differences in their specification and implementation:
1) Correction for deterministic trend:
ADF: In the ADF test, correction for deterministic trend is typically done by including trend terms (linear, quadratic, etc.) in the regression model.
PP: In the PP test, correction for deterministic trend is also done by including trend terms, but the PP test is known to be more robust against heteroscedasticity and autocorrelation issues in the residuals.
2) Correction for Autocorrelated and Heteroscedastic Errors:
ADF: The ADF test uses robust correction of standard errors to account for autocorrelation and heteroscedasticity issues in the residuals.
PP: The PP test uses robust estimation of standard errors for similar reasons.
3) Test Statistic:
ADF: The ADF test statistic is based on the ratio of the coefficients of the unit root in the regression equation.
PP: The PP test statistic is based on the average of t-statistics obtained for the coefficients of the unit root in the regression equation.
4) Asymptotic Distribution:
ADF: The asymptotic distribution of the ADF test statistic depends on the type of underlying process in the time series (e.g., random walk).
PP: The asymptotic distribution of the PP test statistic also depends on specific characteristics of the time series.
5) Test Power:
ADF: The ADF test may be less powerful than the PP test in certain situations, especially when the time series exhibits autocorrelation and heteroscedasticity issues.
PP: The PP test is designed to be more robust against these issues, potentially enhancing its power in certain situations.

In summary, although both tests aim to detect the presence of a unit root in a time series, the PP test is often considered more robust to issues of serial correlation and heteroscedasticity. The choice between the ADF test and the PP test often depends on the specific characteristics of the time series and the assumptions that may be more appropriate in a given context.

- **Kwiatkowski, Phillips, Schmidt, and Shin Test (KPSS)**

The Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test is a stationarity test that assesses whether a time series is stationary around a deterministic trend.

The complete specification of the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test involves several steps. Here is a detailed description of these steps:
1) Determination of the trend: Let Y_t be the observed time series. The first step involves estimating a deterministic trend, usually using linear regression where t is the time variable.

$$Y_t = \alpha + \beta t + \varepsilon_t \tag{10}$$

Here, α and β are the parameters of the regression, and ε_t is the error term.
2) Residual calculation: The residuals u_t are obtained by subtracting the estimated trend from the actual observations.

$$u_t = Y_t - \hat{\alpha} - \hat{\beta} t \tag{11}$$

where $\hat{\alpha}$and$\hat{\beta}$are the estimates of the parameters α et β from the linear regression.
3) Calculation of the $KPSS$ test Statistic: The $KPSS$ test statistic is computed as the weighted sum of squares of the residuals:

$$KPSS = \frac{T \times S_t^2}{\sigma^2} \tag{12}$$

where

S_t^2 is the square of the cumulative sum of the residuals:

$$S_t^2 = \left(\sum_{i=1}^{t} u_i\right)^2 \tag{13}$$

where
T is the sample size.
σ^2 is the estimated variance of the residuals.
4) Choice of the kernel window: The test statistic is affected by a kernel window. The choice of this window can impact the test's power. Different kernel windows can be used, such as the rectangular window or the parabolic window.
5) Determination of critical thresholds: The $KPSS$ test statistic is compared to critical thresholds that depend on the sample size and chosen significance level. These thresholds are often tabulated or obtained through simulations.
6) Decision-making: If the test statistic is greater than the critical threshold, the null hypothesis of stationarity around a deterministic trend is rejected, indicating that the series is non-stationary. Otherwise, the null hypothesis is not rejected.

3.2.2 Normality Tests

Normality tests are statistical procedures used to assess whether a sample of data follows a normal (Gaussian) distribution. These tests are important because many statistical methods assume that data comes from a normal distribution. Here are some of the commonly used normality tests.

- **Jarque-Bera Test:**

The Jarque-Bera test is a statistical test proposed by Jarque and Bera (1980), used to assess whether a sample of data follows a normal distribution. It is based on measures of skewness and kurtosis of the distribution, also known as the third and fourth moments. These measures are used to evaluate if the distribution of data is similar to that of a normal distribution.

Here is how the Jarque-Bera test works:
1) Computation of moments: The third (skewness) and fourth (kurtosis) moments of the sample are calculated.
2) Test statistic: The Jarque-Bera test uses these moments to construct a test statistic that approximately follows a chi-square distribution under the null hypothesis of normality.
3) Comparison to a chi-square distribution: The test statistic is compared to a chi-square distribution with two degrees of freedom (since there are two moments involved). If the test statistic exceeds a critical threshold, the null hypothesis that the data follows a normal distribution is rejected.

In summary, if the Jarque-Bera test statistic is significantly large, it suggests that the sample data is not normally distributed. It is often used in statistical analysis to check for normality adequacy, especially in the context of parametric tests that assume a normal distribution of data. It examines skewness and kurtosis. A large test statistic suggests deviation from normality.

The hypotheses of the Jarque-Bera test are:
H_0 : The series Y_t follows a normal distribution.
H_1 : The series Y_t does not follow a normal distribution.

The Jarque-Bera statistic is expressed as:

$$JB = \frac{T-k}{6}\left(S^2 + \frac{(K-3)^2}{4}\right) \tag{14}$$

T: the number of observations
k: the number of explanatory variables if the data comes from the residuals of a linear regression. Otherwise, k remains zero.
S: the skewness coefficient of the tested sample.
K: the kurtosis of the tested sample.

Mathematically, S and K are defined by:

$$S = \frac{\hat{\mu}_3}{\hat{\sigma}^3} = \frac{\frac{1}{T}\sum_{t=1}^{T}(Y_t - \bar{Y})^3}{\left(\frac{1}{T}\sum_{t=1}^{T}(Y_t - \bar{Y})^2\right)^{3/2}} \tag{15}$$

$$K = \frac{\hat{\mu}_4}{\hat{\sigma}^4} = \frac{\frac{1}{T}\sum_{t=1}^{T}(Y_t - \bar{Y})^4}{\left(\frac{1}{T}\sum_{t=1}^{T}(Y_t - \bar{Y})^2\right)^2} \quad (16)$$

with $\hat{\mu}_3$ and $\hat{\mu}_4$ being the estimators of the third and fourth moments, $\bar{Y}$is the sample mean, and $\hat{\sigma}^2$ is the sample variance.

The JB statistic asymptotically follows a chi-squared distribution with two degrees of freedom.

This test is commonly used to assess whether the residuals of a linear regression follow a normal distribution. Some authors suggest correcting for the number k of regressors, while others do not mention it.

A normal distribution has a skewness coefficient of 0 and a kurtosis of 3. Therefore, if the data follows a normal distribution, the Jarque-Bera test statistic approaches 0, and we accept (do not reject) the null hypothesis H_0 at the significance level α.

A more formal approach:

The Jarque-Bera test does not directly test whether the data follows a normal distribution but rather whether the kurtosis and skewness of the data are the same as those of a normal distribution with the same mean and variance. The hypotheses are as follows:

$H_0 : S = 0 \ and K = 3$

$H_1 : S \neq 0 \ or K \neq 3$

This is a Lagrange multiplier type test.

- **Kolmogorov-Smirnov Test (KS)**

The Kolmogorov-Smirnov test, named after mathematicians Andrey Kolmogorov (Kolmogorov (1933)) and Nikolai Smirnov (Smirnov (1936)), is a statistical method used to assess whether a data sample follows a particular distribution or if two samples come from the same distribution. The Kolmogorov-Smirnov test (KS) for the normal distribution is used to evaluate whether a sample follows a normal distribution.

Here are the steps for implementing the Kolmogorov-Smirnov test for the normal distribution:

1) Formulation of hypotheses:
H_0 (Null Hypothesis): The sample follows a normal distribution.
H_1 (Alternative Hypothesis): There is a significant difference between the empirical distribution of the sample and the normal distribution.
2) Consider a sample of observations $y_1, y_2, \ldots, y_T$of size T.
3) Calculation of the Empirical Cumulative Distribution Function (ECDF):

For each observation y_t in the sample, calculate the proportion of observations less than or equal to y_t. This forms the Empirical Cumulative Distribution Function (ECDF).

$$F_{empirique}\ (y) = \frac{1}{T}\sum_{t=1}^{T} I(y_t \leq y) \quad (17)$$

where $I(.)$ is the indicator function.
4) Estimation of normal distribution parameters:

Estimate the parameters of the normal distribution from the sample, namely the mean μ and the standard deviation σ.
5) Calculation of the cumulative normal distribution function $F_{normal}\ (y)$:

Calculate the cumulative distribution function of a normal distribution with the estimated parametersμ and σ.
6) Calculation of the test statistic:

The KS test statistic (D) is the largest absolute difference between the empirical cumulative distribution $F_{empirical}\ (y)$ and the theoretical cumulative normal distribution:

$$KS = D = \operatorname{Sup}_{y} |F_{empirical}\ (y) - F_{normal}\ (y)| \quad (18)$$

7) Comparison with the critical value:
Compare the test statistic D with the critical value obtained from the Kolmogorov-Smirnov test distribution tables. The larger the value of D, the more divergence there is between the empirical distribution and the normal distribution.
8) Decision-making:

If the KS test statistic is less than the critical value, we do not reject the null hypothesis, indicating that the sample follows the normal distribution.
If the KS test statistic is greater than the critical value, we reject the null hypothesis, suggesting a significant difference between the empirical distribution and the normal distribution.

Additional Remarks:
The chosen significance level (e.g., 0.05) determines the critical value.
Statistical software often provides the test statistic and the associated p-value to facilitate interpretation.
The Kolmogorov-Smirnov test is sensitive to sample size, and other normality tests may also be considered for a more comprehensive evaluation.

- **Lilliefors Test**

The Lilliefors test is a variant of the Kolmogorov-Smirnov (KS) test proposed by Lilliefors (1967) specifically for small-sized samples. It is used to assess whether a sample follows a normal distribution. The Lilliefors test is suitable for situations where the sample size is too small for the standard Kolmogorov-Smirnov test to be reliably applicable. For large sample sizes, the Lilliefors test is equivalent to the Kolmogorov-Smirnov test.

- **Cramer-von Mises Test**

The Cramer-von Mises test is a statistical method used to evaluate the fit of a sample of data to a specified probability distribution. The test is named after statisticians Harald Cramér and Erich von Mises, who contributed to its development. Cramer (1928) published an article in 1928, and von Mises (1928) later made improvements and extensions to the test.

In the case of a normal distribution, the test is used to assess the fit of a sample of data to a theoretical normal distribution. It is designed to detect differences in cumulative distribution functions.

Cramer-von Mises test procedure for the normal distribution:
1) Formulation of hypotheses:
H_0 (Null hypothesis): The sample follows a normal distribution.
H_1 (Alternative hypothesis): There is a significant difference between the empirical distribution of the sample and the normal distribution.
2) Consider a sample of observations $y_1, y_2, \ldots, y_T$ of size T.
3) Calculation of the empirical cumulative distribution function ($ECDF$):
For each observation y_t in the sample, calculate the proportion of observations less than or equal to y_t. This forms the empirical cumulative distribution function ($ECDF$).

$$F_{empirical}(y) = \frac{1}{T}\sum_{t=1}^{T} I(y_t \le y) \quad (19)$$

where $I(.)$ is the indicator function.
4) Estimation of Normal Distribution Parameters:
Estimate the parameters of the normal distribution from the sample, namely the mean μ and the standard deviation σ.
5) Calculation of the Normal Cumulative Distribution Function $F_{normal}(y)$:
Calculate the cumulative distribution function of a normal distribution with the estimated parameters μ and σ.
6) Sorting of Data:
Sort the series of observations $y_1, y_2, \ldots, y_T$ in ascending order, denoted as $y_{(1)}, y_{(2)}, \ldots, y_{(T)}$.
7) Calculation of Test Statistics:
Calculate the Cramer-von Mises test statistic (W^2) using the following formula:

$$W^2 = \frac{1}{12T} + \sum_{t=1}^{T}\left(F_{empiriqie}(y_{(t)}) - \frac{2t-1}{T}\right)^2 \quad (20)$$

8) Comparison with the Critical Value:
Compare the test statistic W^2 with the critical value obtained from specific tables for the Cramer-von Mises test.
9) Decision Making:

- If the test statistic W^2 is less than the critical value, the null hypothesis is not rejected, indicating that the sample follows a normal distribution.
- If the test statistic W^2 is greater than the critical value, the null hypothesis is rejected, suggesting a significant difference between the empirical distribution and the normal distribution.

The Cramer-von Mises test is often used to assess the fit of a distribution when there is a specific interest in detecting differences in the tail of the distribution. As with any statistical test, it is essential to choose the appropriate significance level for decision-making.

- **Anderson-Darling Test**

The Anderson-Darling test, invented in 1952 by Anderson and Darling (1952), is a statistical test used to assess whether a sample of data follows a theoretical distribution, in this case, the normal distribution. It is similar to the Cramer-von Mises test, but it places more emphasis on the tails of the distribution.

1) Formulation of hypotheses:

H_0 (Null Hypothesis): The sample follows a normal distribution.
H_1 (Alternative Hypothesis): There is a significant difference between the empirical distribution of the sample and the normal distribution.
2) Consider a sample of observations $y_1, y_2, \ldots, y_T$ of size T.
3) Calculation of the Empirical Cumulative Distribution Function ($ECDF$):
For each observation y_t in the sample, calculate the proportion of observations less than or equal to y_t. This forms the empirical cumulative distribution function ($ECDF$).

$$F_{empirical}(y) = \frac{1}{T}\sum_{t=1}^{T} I(y_t \leq y) \qquad (21)$$

where $I(.)$ is the indicator function.
4) Estimation of Parameters for the Normal Distribution:
Estimate the parameters of the normal distribution from the sample, namely the mean μ and the standard deviation σ.
5) Calculation of the Normal Cumulative Distribution Function $F_{normal}(y)$:
Calculate the cumulative distribution function of a normal distribution with the estimated parameters μ and σ.
6) Sorting of Data:
Sort the series of observations $y_1, y_2, \ldots, y_T$ in ascending order, denoted as $y_{(1)}, y_{(2)}, \ldots, y_{(T)}$.
7) Calculation of the Anderson-Darling Test Statistic (A^2):

$$A^2 = -T - \frac{1}{T}\sum_{t=}^{T}\Big((2t-1)\ln\big(F_{normale}(y_{(t)})\big) + (2T-2t+1)\ln\big(F_{normale}(y_{(T\mp 1-t)})\big)\Big) \qquad (22)$$

8) Decision Making:

If the test statistic A^2 is less than the critical value, the null hypothesis is not rejected, indicating that the sample follows a normal distribution.

If the test statistic A^2 is greater than the critical value, the null hypothesis is rejected, suggesting a significant difference between the empirical distribution and the normal distribution.

The Anderson-Darling test is sensitive to the tails of the distribution, making it particularly useful when detecting deviations in these regions. As with any statistical test, it is important to choose the appropriate significance level for decision-making.

- **Quantile-Quantile Plots or QQ Plots**

Quantile-Quantile plots (QQ plots) are powerful visual tools for assessing whether a sample of data follows a normal distribution.

A Quantile-Quantile plot for the normal distribution can be formulated mathematically using the empirical quantiles of the sample and the theoretical quantiles of the normal distribution. Here is how it can be expressed mathematically:

Consider a sample of observations $y_1, y_2, \ldots, y_T$ of size T. Arrange the series in ascending order: $y_{(1)}$, $y_{(2)}, \ldots, y_{(T)}$.

The empirical quantiles of the sample are defined as follows: for $t = 1,2,\ldots,T$

$$Q_t(empirical) = \frac{t - 0,5}{T} \tag{23}$$

The theoretical quantiles for a normal distribution are given by the inverse cumulative distribution function (the quantile function) of the normal distribution. For a mean μ and a standard deviation σ, the theoretical quantiles of the normal distribution are defined as:

$$Q_t(normale) = \mu + \sigma . ppf\left(Q_t(empirique)\right)$$

where ppf is the inverse percent point function (quantile function) of the normal distribution.

The QQ plot can then be depicted by placing theoretical quantiles on the horizontal axis and empirical quantiles on the vertical axis. The mathematical formula for a specific point on the QQ plot is therefore $\left(Q_t(normal), Q_t(empirical)\right)$ for $= 1,2,\ldots,T$.

The main idea of a QQ plot is to compare the empirical quantiles of the sample with the theoretical quantiles of a normal distribution. A perfect alignment along the 45-degree line (the first bisector) suggests a perfect fit to the normal distribution.

If the points are above the 45-degree line, it indicates that the sample values are higher than expected according to a normal distribution. If the points are below the 45-degree line, it indicates that the sample values are lower than expected according to a normal distribution.

The overall shape of the curve can provide insights into the distribution of deviations from normality. For example, an S-shaped curve may indicate a heavier or lighter tail distribution than expected.

Extreme points on the edges of the QQ plot can be particularly informative as they indicate potential divergences in the tails of the distribution.

3.2.3 Autocorrelation Function or Serial Correlation: Ljung-Box Test

The Ljung-Box Q test is a statistical method proposed by Ljung and Box (1978) used in time series analysis to assess whether the residuals of a time series model exhibit significant autocorrelation at different lags. The Ljung-Box Q test is an extension of the Box-Pierce autocorrelation test.

This is a parametric test that determines the serial correlation ρ_k, the autocorrelation between current returns r_t and previous returns r_{t-k} in the same series. If autocorrelation in the returns series is detected (positive or negative), one can conclude that the returns series does not behave randomly, indicating a weak-form inefficiency in the stock market.

The serial correlation test determines whether correlation coefficients differ significantly from zero by measuring the correlation coefficient between the returns of the series and delayed returns in the same series.

The serial correlation coefficient of a random variable Y_t for a lag q is defined as:

$$\rho_q = \frac{Cov(Y_t, Y_{t-k})}{\sqrt{Var(Y_t).Var(Y_{t-k})}} \tag{24}$$

$Var(Y_t)$ is the variance of Y_t defined by:

$$Var(Y_t) = E\left(\left(Y_t - E(Y_t)\right)^2\right) \tag{25}$$

$Cov(Y_t, Y_{t-q})$ is the covariance between Y_t and Y_{t-q} defined by:

$$Cov(Y_t, Y_{t-q}) = E\left(\left(Y_t - E(Y_t)\right)\left(Y_{t-q} - E(Y_{t-q})\right)\right) \tag{26}$$

Serial correlation can be estimated using the empirical autocorrelation coefficient at lag q, given as follows:

$$\hat{\rho}_q = \frac{\sum_{t=q+1}^{T}(Y_t - \bar{Y})(Y_{t-q} - \bar{Y})}{\sum_{t=1}^{T}(Y_t - \bar{Y})^2} \tag{27}$$

where $\bar{Y}$ is the empirical mean of Y_t defined by:

$$\bar{Y} = \frac{1}{T}\sum_{t=1}^{T} Y_t \tag{28}$$

The Q statistic at lag q is a test statistic for the null hypothesis that there is no autocorrelation up to order q and is calculated as follows:

$$Q_{LB} = T(T+2)\sum_{j=1}^{q}\frac{\rho_j^2}{T-j} \quad (29)$$

Q_{LB} follows a chi-square distribution with q degrees of freedom. Using this test statistic, the null and alternative hypotheses tested were as follows:

H_0 : All autocorrelations ρ_j are zero up to lag q, or $\forall j : \rho_j = 0$

H_1 : At least one autocorrelation ρ_j up to lag j, or $\exists j$ such that $\rho_j \neq 0$

Given the obtained value of Q_{LB}, the null hypothesis that all autocorrelations up to lag k are zero will be rejected if the statistic exceeds the critical value Q in the chi-square table (Gujarat 2004). Alternatively, the $P-value$ can be used to test the hypothesis. The null hypothesis of all zero autocorrelations can be rejected if the $P-value$ obtained from the statistical test is less than the chosen significance level.

3.2.4 Variance Ratio Test

The variance ratio test, suggested by Lo and MacKinlay (1988), is a frequently used test to examine the presence of a random walk in stock returns. This test considers an important characteristic in the random walk process, namely that the variance of returns over a q-period horizon is equal to q times the variance of returns over a one-period horizon. In other words, returns are independently and identically distributed with a constant mean and a variance that is a linear function of the holding period.

The null and alternative hypotheses for the variance ratio test are as follows:

H_0: The returns series follows a random walk.

H_a: The returns series does not follow a random walk.

We denote:

p_t : the index value at date t

$P_t = ln\ (p_t)$: the logarithmic index value at date t

$r_t = P_t - P_{t-1}$: the geometric return of the index over the interval $[t-1, t]$

$R_t = 100.\,ln\ \left(\frac{p_t}{p_{t-1}}\right)$: the percentage geometric return of the index over the interval $[t-1, t]$

The hypothesis of a random walk is expressed as follows:

$$P_t = \mu + P_{t-1} + \varepsilon_t \quad (30)$$

where ε_t is the error term with $E(\varepsilon_t) = 0$ and $Var\ (\varepsilon_t) = \sigma_0^2$. The constant μ represents the mathematical expectation of $r_t = P_t - P_{t-1}$.

We assume that we have $nq + 1$ observations $P_0, P_1, \ldots, P_{nq}$ from the log-price series P_t, where n and q are integers greater than 1.

Lo and MacKinlay (1988) define the estimators for the unknown parameters μ and σ_0^2:

$$\hat{\mu} = \frac{1}{nq}\sum_{j=1}^{nq}(P_j - P_{j-1}) = \frac{1}{nq}(P_{nq} - P_0) \quad (31)$$

$$\hat{\sigma}^2(1) = \frac{1}{nq}\sum_{j=1}^{nq}(P_j - P_{j-1} - \hat{\mu})^2 \quad (32)$$

$$\hat{\sigma}^2(q) = \frac{1}{nq^2}\sum_{j=q}^{nq}(P_j - P_{j-q} - q\hat{\mu})^2 \quad (33)$$

Lo and MacKinlay's (1988) variance ratio test considers that under the null hypothesis of a random walk, the variance of one-period differences is equal to q times the variance of q-period differences. The chosen test statistic is given by:

$$VR\ (q) = \frac{\hat{\sigma}^2(q)}{\hat{\sigma}^2(1)} \quad (34)$$

Under the null hypothesis, the $VR\ (q)$statistic is equal to 1.

Lo and MacKinlay (1988) have defined two test statistics, $Z(q)$) and $Z^*(q)$, respectively under the assumptions of homoscedasticity and heteroscedasticity.

$$Z(q) = \frac{VR\ (q) - 1}{\sqrt{\theta(q)}} \to \mathcal{N}(0,1) \quad (35)$$

$$Z^*(q) = \frac{VR(q) - 1}{\sqrt{\theta^*(q)}} \to \mathcal{N}(0,1) \tag{36}$$

where

$$\theta(q) = \frac{2(2q-1)(q-1)}{3nq^2} \tag{37}$$

$$\theta^*(q) = \sum_{j=1}^{q-1} \left(\frac{2(q-j)}{q}\right)^2 \delta(j) \tag{38}$$

$$\delta(j) = \frac{\sum_{t=j+1}^{nq}(P_t - P_{t-1} - \hat{\mu})^2 (P_{t-j} - P_{t-j-1} - \hat{\mu})^2}{(\sum_{t=1}^{nq}(P_t - P_{t-1} - \hat{\mu})^2)^2} \tag{39}$$

If $|Z(q)| > Z_{1-\alpha/2}$, then we reject the null hypothesis of a random walk under homoscedasticity at the significance level of α %. This is equivalent to saying that the $p - value \quad = p\left(|Z| > Z_{1-\alpha/2}\right)$ is greater than α.

If $|Z^*(q)| > Z_{1-\alpha/2}$, then we reject the null hypothesis of a random walk under heteroscedasticity at the significance level of α %. This is equivalent to saying that the $p - value \quad = p\left(|Z| > Z_{1-\alpha/2}\right)$ is greater than α.

Most statistical software uses the threshold $\alpha = 5\%$, and we reject the hypothesis that the returns series follows a random walk if the $p - value \quad > 1.96$.

If the null hypothesis of a random walk and the variance ratio $VR(q)$ is greater than 1, it implies that the returns series is positively correlated, and if the variance ratio is less than 1, it suggests that the returns series is negatively correlated.

3.2.5 Runs Test

The Runs Test, also known as the Sequence Test, was suggested by Wald and Wolfowitz (1940) to assess whether a sequence of data follows a random model. It is a non-parametric test widely applied to examine whether successive price changes are independent.

Stock returns can be negative, positive, or zero. Each return in a time series is assigned the sign +, -, or 0. A run can be defined as a sequence of the same sign preceded and followed by a sequence of a different sign. The number of runs is the count of occurrences of different successive runs. For example, in the time series of signs ++, --, 0 0++++0----, we count 6 runs.

The Runs Test was the most commonly used non-parametric test for testing the hypothesis of a random walk. It does not require the series of returns to be normally or identically distributed, a condition often difficult to satisfy for most stock return statistics. At the same time, it eliminates the effect of extreme values often present in return series. This provides a robust alternative to parametric serial correlation tests where it is assumed that distributions are normally distributed.

The null hypothesis of randomness is tested by observing the number of runs. The total number of observed runs, denoted as R, is the sum of the number of positive runs, negative runs, and zero runs. In the case where asset price changes are positively correlated, one should observe long positive or negative runs (each run contains a significant number of identical signs). Conversely, if price changes are negatively correlated, short runs, i.e., frequent sign changes, should be observed. If changes are independent, neither of these cases should be observed.

In an efficient market, series are uncorrelated (randomly distributed), the signs of price changes are randomly distributed, the number of positive runs and the number of negative runs are approximately equal, and the total number of runs follows a normal distribution for which we can calculate the mean and standard deviation.

The test is based on the idea that if a data series is random, the observed number of runs in the series should be close to the expected number of runs.

The null and alternative hypotheses of the runs test are as follows:

H_0: The returns series follows a random walk (independence).

H_a: The returns series does not follow a random walk.

Under the null hypothesis and for samples of size $N \geq 30$, the total number of runs R follows a normal distribution with the mathematical expectation $E(R)$ and standard deviation σ_R approximated by:

$$E(R) = \frac{N(N+1) - \sum_{i=1}^{3} n_i^2}{N} \quad (40)$$

$$\sigma_R = \left(\frac{\sum_{i=1}^{3} n_i^2 \left(\sum_{i=1}^{3} n_i^2 + N(N+1) \right) - 2N\sum_{i=1}^{3} n_i^3 - N^3}{N^2(N-1)} \right)^{\frac{1}{2}} \quad (41)$$

where:

N_1 : the number of + signs in the time series of signs associated with the returns series.
N_2 : the number of - signs in the time series of signs associated with the returns series.
N_3 : the number of 0 signs in the time series of signs associated with the returns series.
$N = \sum_{i=1}^{3} N_i$: the total number of +, -, and 0 signs in the time series of signs associated with the returns series.

To implement the Runs Test, the reduced statistic Z is used and defined as:

$$Z = \frac{R - E(R)}{\sigma_R} \quad (42)$$

Under the null hypothesis, we have $Z \to \mathcal{N}(0,1)$.

Note that some authors use alternative formulas for $E(R)$ and σ_R:

$$E(R) = \frac{2N_1N_2 + N}{N} \quad (43)$$

$$\sigma_R = \left(\frac{2N_1N_2(2N_1N_2 - N)}{N^2(N-1)} \right)^{\frac{1}{2}} \quad (44)$$

where:

N_1 : the number of + signs in the time series of signs associated with the returns series.
N_2 : the number of - and 0 signs in the time series of signs associated with the returns series.

We can deduce these two previous formulas by taking $N_3 = 0$ in the formulas (40) and (41). In our case, we will use these two formulas because all returns of the MASI index are non-zero in the studied period.

Decision rule:

We set a significance level $\alpha = 1\%$ or 5% or 10%.

The Runs Test is a two-tailed test, which involves comparing the calculated value of the Z statistic to the critical value $Z_{1-\alpha/2}$ (= 2,58 or 1,96 or 1,65).

If $|Z| > Z_{1-\alpha/2}$, then we reject the null hypothesis at the significance level $\alpha\ \%$. This means that the $p - value \quad = p(|Z| > Z_{1-\alpha/2})$ is greater than α. We conclude that the returns series does not follow a random walk (inefficiency of the stock market).

Most statistical software uses a threshold of $\alpha = 5\%$, and we reject the hypothesis that the returns series follows a random walk if the si $p - value \quad > 1{,}96$.

4. Results and Discussion

In this section, we will apply the various tests mentioned in the introduction to test the hypothesis of a random walk on the Moroccan stock market represented by the MASI index. The data used in this section are the daily closing prices of the Casablanca Stock Exchange index (MASI), covering the period from 03/01/2002 to 23/01/2024, totaling 5502observations.

4.1 Price and Geometric Return Plots of the MASI Index

Figures 1 and 2 below describe the plots of MASI Index Prices and geometric returns of MASI Index.

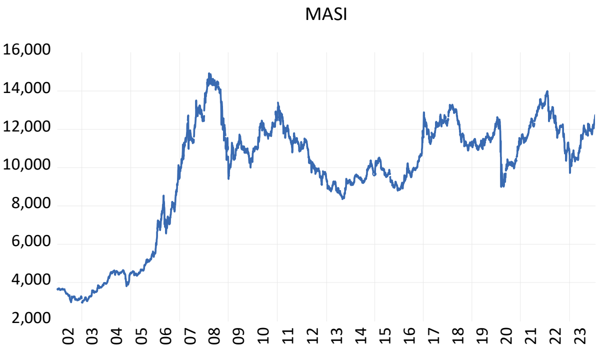

Figure 1: Plot of MASI Index Prices

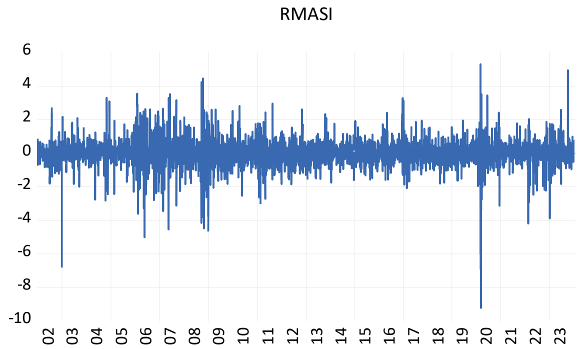

Figure 2: Plot of Geometric Returns of MASI Index

4.2 Histogram and Descriptive Statistics of Geometric Returns Series

The figure 3 below shows the histogram and descriptive statistics of the geometric returns of the MASI index during the period from 03/01/2002 to 22/01/2024.

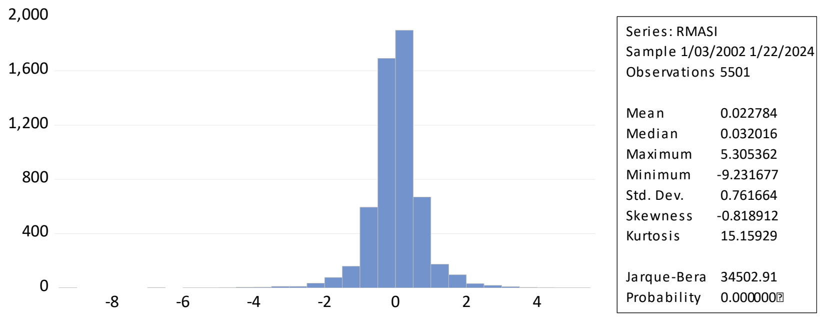

Figure 3: Histogram and Descriptive Statistics of the Geometric Returns Series of the MASI Index

We observe that the average and median returns are positive during the study period. The returns exhibit a negative skewness coefficient different from 0. The kurtosis coefficient is high, exceeding three. Negative skewness and high kurtosis indicate a significant deviation from normality in the unconditional distribution of returns.

The Jarque-Bera statistic rejects the hypothesis of a normal distribution of returns at a significance level of 1%. Therefore, we can reject the null hypothesis of normality in the level series for the MASI index returns.

4.3 Stationarity Tests (Unit Root)

Standard tests used in econometrics to identify whether a process is stationary or not include the tests by Dickey and Fuller (1979), Philips-Perron (1995), and Kwiatkowski et al. (1992). We applied these three tests to the series of geometric returns of the MASI index.

4.3.1 Augmented Dickey Fuller Test (ADF)

We applied the Augmented Dickey Fuller test to the series of geometric returns of the MASI index in its three forms: ADF without a constant and trend, ADF with a constant, and ADF with a constant and trend. The results are provided in the tables 1, 2 and 3.

Null Hypothesis: RMASI has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=32)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-57.62284	0.0001
Test critical values:	1% level	-2.565383	
	5% level	-1.940882	

	10% level	-1.616661	

*MacKinnon (1996) one-sided p-values.

Table 1: Result of the ADF test without a constant and without trend

Null Hypothesis: RMASI has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=32)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-57.65826	0.0001
Test critical values:	1% level	-3.431358	
	5% level	-2.861870	
	10% level	-2.566988	

*MacKinnon (1996) one-sided p-values.

Table 2: Result of the ADF test with a constant

Null Hypothesis: RMASI has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=32)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-57.67746	0.0000
Test critical values:	1% level	-3.959706	
	5% level	-3.410621	
	10% level	-3.127089	

*MacKinnon (1996) one-sided p-values.

Table 3: Result of the ADF test with a constant and with a trend

We can observe that the results of the ADF unit root tests indicate that the t-statistic for all three tests is well below the critical values at the 1%, 5%, and 10% significance levels. This implies that the series of geometric returns of the MASI index is stationary.

4.3.2 Philips-Perron Test (PP)

We applied the Philips-Perron test to the series of geometric returns of the MASI index in its three forms: PP without a constant and without trend, PP with a constant, and PP with a constant and trend. The results are provided in the tables 4,5 and 6.

Null Hypothesis: RMASI has a unit root
Exogenous: None
Bandwidth: 2 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-57.67940	0.0001
Test critical values:	1% level	-2.565383	
	5% level	-1.940882	
	10% level	-1.616661	

*MacKinnon (1996) one-sided p-values.

Table 4: Result of the Philips-Perron test without a constant and without trend

Null Hypothesis: RMASI has a unit root
Exogenous: Constant
Bandwidth: 0 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-57.65826	0.0001
Test critical values:	1% level	-3.431358	
	5% level	-2.861870	
	10% level	-2.566988	

*MacKinnon (1996) one-sided p-values.

Table 5: Result of the Philips-Perron test with a constant

Null Hypothesis: RMASI has a unit root
Exogenous: Constant, Linear Trend
Bandwidth: 2 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-57.73309	0.0000
Test critical values:	1% level	-3.959706	
	5% level	-3.410621	
	10% level	-3.127089	

*MacKinnon (1996) one-sided p-values.

Table 6: Result of the Philips-Perron test with a constant and with a trend

We can observe that the results of the Philips-Perron tests indicate that the t-statistic for all three tests is well below the critical values at the 1%, 5%, and 10% significance levels. This implies that the series of geometric returns of the MASI index is stationary.

4.3.3 Kwiatkowski-Philips-Schmidt-Shin (KPSS) Test

We applied the Kwiatkowski-Philips-Schmidt-Shin test to the series of geometric returns of the MASI index in its three forms: KPSS with a constant and KPSS with a constant and trend. The results are provided in the tables 7 and 8.

Null Hypothesis: RMASI is stationary
Exogenous: Constant
Bandwidth: 13 (Newey-West automatic) using Bartlett kernel

		LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic		0.348995
Asymptotic critical values*:	1% level	0.739000
	5% level	0.463000
	10% level	0.347000

*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)

Residual variance (no correction)	0.580027
HAC corrected variance (Bartlett kernel)	0.921760

Table 7: Result of the KPSS test with a constant

Null Hypothesis: RMASI is stationary
Exogenous: Constant, Linear Trend
Bandwidth: 13 (Newey-West automatic) using Bartlett kernel

		LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic		0.120142
Asymptotic critical values*:	1% level	0.216000
	5% level	0.146000
	10% level	0.119000

*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)

Residual variance (no correction)	0.579708
HAC corrected variance (Bartlett kernel)	0.917211

Table 8: Result of the KPSS test with a constant and with a trend

We can observe that the results of both KPSS tests indicate that the t-statistic for both tests is well below the critical values at the 1% and 5% significance levels. This implies that the series of geometric returns of the MASI index is stationary.

4.4 Normality Tests

4.4.1 Jarque-Bera Test

The application of the Jarque-Bera test yields the following result (table 9).

Series: RMASI
Sample 1/03/2002 1/22/2024
Observations 5501

Mean	0.022784
Median	0.032016
Maximum	5.305362
Minimum	-9.231677
Std. Dev.	0.761664
Skewness	-0.818912
Kurtosis	15.15929
Jarque-Bera	34502.91
Probability	0.000000

Table 9: Some statistics and Jarque-Bera results for geometric returns of MASI

The JB statistic asymptotically follows a chi-squared distribution with two degrees of freedom. The p-value of the JB statistic being less than 5% leads to the rejection of the null hypothesis that skewness is zero and kurtosis is equal to 3, consequently rejecting the hypothesis of normality for the series of geometric returns.

4.4.2 Quantile-Quantile Plot (QQ plot)

We use the Quantile-Quantile plot (QQ-Plot) test to check if the geometric return of MASI follows a normal distribution. The QQ-Plot should lie on a straight line at 45 degrees if the empirical distribution and the theoretical (normal) distribution are the same. The figure 4 below shows the QQ-Plot of the empirical distribution of the geometric return of MASI.

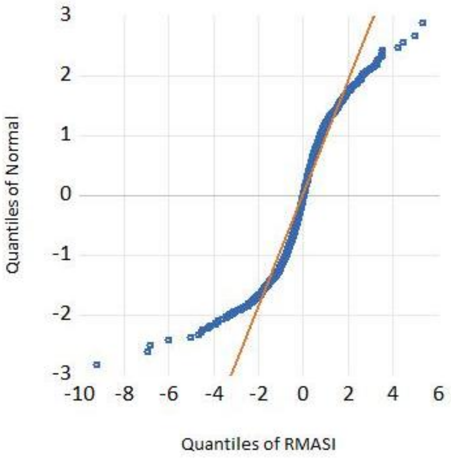

Figure 4: QQ-Plot of the Geometric Return of MASI

The examination of Figure 4 reveals that the geometric return of MASI from the empirical distribution indicates a deviation from normality with thicker tails than those of the normal distribution. The QQ-Plot is not on a straight line and exhibits an S-shape. This confirms the earlier result from the Jarque-Bera (JB) statistics.

4.4.3 Lilliefors, Cramer-von Mises, and Anderson-Darling Tests

We applied the Lilliefors, Cramer-von Mises, and Anderson-Darling tests to test the normality hypothesis of the series of geometric returns of the MASI index. The results of these tests are provided in the table 10.

Empirical Distribution Test for RMASI
Hypothesis: Normal
Date: 01/24/24 Time: 04:49
Sample: 1/03/2002 1/22/2024
Included observations: 5501

Method	Value	Adj. Value	Probability
Lilliefors (D)	0.087942	NA	0.0000
Cramer-von Mises (W2)	19.02265	19.02438	0.0000
Watson (U2)	18.98164	18.98336	0.0000
Anderson-Darling (A2)	110.3603	110.3753	0.0000

Method: Maximum Likelihood - d.f. corrected (Exact Solution)

Parameter	Value	Std. Error	z-Statistic	Prob.
MU	0.022784	0.010269	2.218621	0.0265
SIGMA	0.761664	0.007262	104.8809	0.0000

Log likelihood	-6307.437	Mean dependent var.	0.022784
No. of Coefficients	2	S.D. dependent var.	0.761664

Table 10: Lilliefors, Cramer-von Mises, and Anderson-Darling Normality Tests

We observe that all p-values from the 3 tests are below the 5% significance threshold, indicating that the statistics of all 3 tests exceed the critical value at the 5% level. This implies that the null hypothesis of normality can be rejected at the 5% threshold for all three tests.

We note that the parameters of the estimated normal distribution are significant at 5% and are equal to:

The mean $\mu = 0.022784$

The standard deviation $\sigma = 0.761664$

We will now repeat the same tests by imposing the mean and standard deviation of the estimated distribution to be equal to the previously estimated values.

We obtain the following results (table 11).

Empirical Distribution Test for RMASI
Hypothesis: Normal
Date: 01/24/24 Time: 04:51
Sample: 1/03/2002 1/22/2024
Included observations: 5501

Method	Value	Adj. Value	Probability
Kolmogorov (D+)	0.085826	6.376056	0.0000
Kolmogorov (D-)	0.087942	6.533202	0.0000
Kolmogorov (D)	0.087942	6.533202	0.0000
Kuiper (V)	0.173768	12.91564	0.0000
Cramer-von Mises (W2)	19.02262	19.02600	0.0000
Watson (U2)	18.98161	18.98435	0.0000
Anderson-Darling (A2)	110.3601	110.3601	0.0000

Parameter	Value	Std. Error	z-Statistic	Prob.
MU	0.022784	*	NA	NA
SIGMA	0.761664	*	NA	NA
Log likelihood	-6307.437	Mean dependent var.		0.022784
No. of Coefficients	0	S.D. dependent var.		0.761664

* Fixed parameter value

Table 11:Normality Tests with $\mu = 0.022784$ and $\sigma = 0.761664$

We can observe that all tests display p-values below 5%, implying a rejection of the normality hypothesis.

4.5 Ljung-Box Autocorrelation Test

The table 12 below presents the autocorrelation and partial correlation functions of the MASI index returns with a lag of 36.

Date: 01/24/24 Time: 04:47
Sample: 1/03/2002 1/22/2024
Included observations: 5501

Lag	AC	PAC	Q-Stat	Prob
1	0.246	0.246	334.34	0.000
2	0.068	0.008	360.15	0.000
3	-0.011	-0.031	360.77	0.000
4	-0.010	-0.001	361.37	0.000
5	0.013	0.020	362.34	0.000
6	-0.016	-0.025	363.76	0.000
7	0.009	0.018	364.16	0.000
8	-0.012	-0.016	364.94	0.000
9	0.022	0.029	367.59	0.000
10	0.028	0.018	371.82	0.000
11	0.013	0.000	372.72	0.000
12	0.030	0.026	377.54	0.000
13	0.023	0.013	380.37	0.000
14	0.023	0.012	383.35	0.000
15	0.034	0.028	389.72	0.000
16	-0.002	-0.018	389.73	0.000
17	-0.001	0.002	389.74	0.000
18	-0.003	-0.000	389.79	0.000
19	0.004	0.004	389.89	0.000
20	0.018	0.017	391.76	0.000
21	0.011	0.003	392.47	0.000
22	-0.011	-0.020	393.15	0.000
23	-0.025	-0.018	396.53	0.000
24	0.014	0.024	397.54	0.000
25	0.020	0.011	399.78	0.000
26	0.028	0.019	404.24	0.000
27	0.017	0.004	405.93	0.000
28	0.027	0.023	409.97	0.000
29	0.019	0.006	411.97	0.000
30	0.010	0.002	412.53	0.000
31	-0.023	-0.028	415.33	0.000
32	-0.024	-0.010	418.45	0.000
33	-0.039	-0.032	426.99	0.000
34	-0.046	-0.031	438.50	0.000
35	-0.046	-0.030	450.14	0.000
36	-0.048	-0.032	463.14	0.000

Table 12: Ljung-Box Autocorrelation Test with a lag of 36

The Ljung-Box test was conducted with 36 lags. The results show that the p-values for all lags are below alpha (0.05), indicating that the test is statistically significant at the 5% level. Therefore, the null hypothesis that "all autocorrelations up to 36 lags are zero" is rejected in favor of the alternative hypothesis.

These results lead us to reject the null hypothesis of no serial autocorrelation and accept the alternative hypothesis that the studied time series exhibits serial dependence, thereby rejecting the weak-form efficiency hypothesis of the Moroccan stock market.

4.6 Variance Ratio Test

The results of the variance ratio test under both hypotheses, namely heteroscedasticity and homoscedasticity, are presented in tables 13 and 14 respectively.

Null Hypothesis: RMASI is a martingale
Date: 01/24/24 Time: 04:54
Sample: 1/03/2002 1/22/2024
Included observations: 5500 (after adjustments)
Heteroskedasticity robust standard error estimates
User-specified lags: 2 4 8 16

Joint Tests	Value	df	Probability
Max |z| (at period 2)*	11.34817	5500	0.0000

Individual Tests

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.618336	0.033632	-11.34817	0.0000
4	0.335579	0.060589	-10.96603	0.0000

8	0.168231	0.090529	-9.187903	0.0000
16	0.083488	0.120951	-7.577570	0.0000

*Probability approximation using studentized maximum modulus with parameter value 4 and infinite degrees of freedom

Test Details (Mean = -0.000118671518389)

Period	Variance	Var. Ratio	Obs.
1	0.87433	--	5500
2	0.54063	0.61834	5499
4	0.29341	0.33558	5497
8	0.14709	0.16823	5493
16	0.07300	0.08349	5485

Table 13: Heteroscedasticity Variance Ratio Test

Null Hypothesis: RMASI is a random walk
Date: 01/24/24 Time: 04:55
Sample: 1/03/2002 1/22/2024
Included observations: 5500 (after adjustments)
Standard error estimates assume no heteroskedasticity
User-specified lags: 2 4 8 16

Joint Tests	Value	df	Probability
Max |z| (at period 2)*	28.30498	5500	0.0000
Wald (Chi-Square)	838.5068	4	0.0000

Individual Tests

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.618336	0.013484	-28.30498	0.0000
4	0.335579	0.025226	-26.33846	0.0000
8	0.168231	0.039886	-20.85356	0.0000
16	0.083488	0.059353	-15.44183	0.0000

*Probability approximation using studentized maximum modulus with parameter value 4 and infinite degrees of freedom

Test Details (Mean = -0.000118671518389)

Period	Variance	Var. Ratio	Obs.
1	0.87433	--	5500
2	0.54063	0.61834	5499
4	0.29341	0.33558	5497
8	0.14709	0.16823	5493
16	0.07300	0.08349	5485

Table 14: Homoscedasticity Variance Ratio Test

Results of the variance ratio test (Lo and Macklay 1988) under the heteroscedasticity assumption, as displayed in Table 13, indicate that the p-value for the joint test is less than 0.05, demonstrating statistical significance at a 5% level. This implies rejecting the null hypothesis that the series of geometric returns follows a random walk. Similarly, the individual test for all periods (2, 4, 8, 16) strongly rejects the null hypothesis of a random walk, as the p-values are below the 5% significance threshold.

Under the homoscedasticity assumption (Table 14), the results indicate that the joint tests, i.e., tests for the joint null hypothesis across all periods, strongly reject the null hypothesis of a random walk, with a p-value of 0.0000. Regarding individual tests, i.e., the test for each individual period (periods 2, 4, 10, and 16), the p-values for all periods (0.0000) are below 0.05, also leading to the rejection of the null hypothesis of a random walk under the homoscedasticity assumption.

Based on the results of the variance ratio test, it can be concluded that the series of geometric returns of the MASI does not behave randomly. Therefore, the Moroccan stock market is not efficient in terms of weak form efficiency.

4.7 Runs Test

This is a non-parametric test used to examine whether the series of geometric returns of the MASI behaves randomly. The results of the Runs Test are presented in the table 15 below.

N_1	2902
N_2	2599
N	5501
R	2425
$E(R)$	2743,155
$Var\ (R)$	1366,668
σ_R	36,968
Z	-8,606
$p-value$ (Asymp. Sig. (2-tailed))	0,000
α	0,050
$1-\alpha/2$	0,975
$z_{1-\alpha/2}$	1,960

Table 15: Results of the Runs Test

As shown in Table 15, the value of the Z-statistic is negative (-8,606), indicating that the number of observed Runs is less than the expected number of Runs. This suggests the presence of positive serial correlation in the daily returns series (i.e., the series does not behave randomly). Additionally, the obtained p-value (0.000) is less than 5%, indicating that the test is statistically significant at the 5% level. Therefore, the null hypothesis that the series is random is rejected in favor of the alternative hypothesis.

Based on these results, the conclusion drawn from the Runs Test is that the geometric returns series of the MASI does not behave randomly, suggesting that the Moroccan stock market is not efficient in terms of weak form efficiency.

We conclude that all the applied tests led to the same results, indicating that the geometric returns series of the MASI does not follow a random walk, and consequently, the Moroccan stock market is inefficient in its weak form.

Our findings align perfectly with the results of other studies analyzing the informational efficiency of the stock market during different periods. Notably, the study by Chiny and Mir (2015), which examined daily prices of the MASI and three other sectoral indices from January 2002 to December 2013, formally rejected the weak form efficiency hypothesis for both the overall market represented by the MASI and other sectoral markets.

In their study, Inayat FAITEH and Amal NAJAB (2020) also concluded the weak inefficiency of the Moroccan stock market. Similarly, the results of tests conducted by Abdelhadi Ifleh and Mounime El Kabbouri (2021) for the period from January 2012 to January 2021 questioned the weak form efficiency of the Moroccan stock market.

However, it is worth noting that the study by Khalid Abouahmed (2019) is the only one contradicting these results. Examining the informational efficiency of the Moroccan stock market from January 2013 to December 2015, the author concluded, based on normality tests and unit root tests, that the monthly returns series of the MADEX follows a normal distribution and is stationary, suggesting the informational efficiency of the Moroccan stock market during that period. It is evident that Abouahmed (2019) drew inconclusive conclusions by relying solely on the application of these two types of tests.

5. Conclusion

This study examined the weak form efficiency of the Moroccan stock market based on the random walk hypothesis. We analyzed the daily geometric returns of the MASI index, representing the Casablanca stock market, over the period from 03/01/2002 to 22/01/2024. To assess the random walk hypothesis (RWH), various tests, including unit root tests, normality tests, the Ljung-Box serial correlation test, the variance ratio test, and the Runs test, were applied.

The results of the analyses concerning the stationarity of the series of daily geometric returns of the MASI index, obtained from unit root tests such as Augmented Dickey-Fuller (ADF), Philips-Perron (PP), and Kwiatkowski-Philips-Schmidt-Shin (KPSS), confirm the stationary nature of the series. This finding contradicts the random walk hypothesis for the returns of the MASI index during the observation period. Therefore, it follows that the Moroccan stock market exhibits inefficiency in its weak form.

We also evaluated the normality of the series of daily geometric returns of the MASI index by subjecting it to a battery of tests.

The conclusions of the Jarque-Bera test invalidated the null hypothesis stating that skewness is zero and kurtosis is equal to 3. Consequently, the hypothesis of normality for the series of geometric returns was refuted.

Examination of the Quantile-Quantile (Q-Q plot) diagram revealed a deviation of the empirical distribution of the geometric return of the MASI index from normality, characterized by thicker tails than those of a normal distribution. The Q-Q plot does not align with a straight line but exhibits an S-shaped configuration. This observation confirms that the series of geometric returns of the MASI index does not follow a normal distribution, leading us to conclude the inefficiency of the Moroccan stock market in its weak form.

We also subjected the series of geometric returns of the MASI index to Kolmogorov-Smirnov (Lilliefors), Cramer-von Mise, and Anderson-Darling tests to verify the normality hypothesis. The results of these tests indicate that the null hypothesis of normality can be rejected at the 5% level for all three tests. This finding confirms the inefficiency of the Moroccan stock market.

The execution of the Ljung-Box autocorrelation test over 36 lags resulted in the rejection of the null hypothesis stipulating the absence of serial autocorrelation. Instead, the acceptance of the alternative hypothesis suggests that the series of geometric returns of the MASI index exhibits serial dependence. Therefore, this leads to the rejection of the weak form efficiency hypothesis of the Moroccan stock market.

The conclusions from the variance ratio test, performed under the assumptions of heteroscedasticity and homoscedasticity, led to the rejection of the null hypothesis stating that the series of geometric returns follows a random walk. Consequently, it follows that the Moroccan stock market does not exhibit efficiency in its weak form.

The conclusions from the Runs test also led to the rejection of the random walk hypothesis, indicating that the Moroccan stock market does not exhibit efficiency in its weak form.

In conclusion, the consistent results obtained from all applied tests indicate that the series of geometric returns of the MASI index does not follow a random walk. Therefore, it is established that the Moroccan stock market exhibits inefficiencies in its weak form.

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