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Assessing weak-form efficiency in the Moroccan stock market: A focus on ARFIMA models

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Abstract: Objective: The objective of this study is to assess the weak form of informational efficiency in the Moroccan stock market, utilizing the MASI index (Moroccan All Shares Index).
Methodology: For this purpose, we utilized the Autoregressive Fractionally Integrated Moving Average (ARFIMA) model on the daily geometric returns of the MASI index. The analysis spans from March 1, 2002, to January 22, 2024, encompassing a total of 5501 observations.
Results: The obtained results suggest that the series of geometric returns for the MASI index displays characteristics of long memory.
Conclusion: We concluded that the Moroccan stock market is inefficient in its weak form, implying the potential for investors to attain abnormal returns.
Keywords: Weak form informational efficiency, long memory, ARFIMA model, MASI index.
JEL Classification: C12, C13, C22, G11, G14

1. Introduction

Proposed by economist Eugene Fama (1965) in the 1960s, the Efficient Market Hypothesis (*EMH*) asserts the efficiency of financial markets. This concept implies that financial asset prices reflect all available information, making it difficult to attain extraordinary profits through fundamental or technical analysis. Various studies have been undertaken to empirically substantiate the Efficient Market Hypothesis (*EMH*).

In 1970, Fama (1970) classified efficient markets into three categories: weak-form efficiency, semi-strong-form efficiency, and strong-form efficiency:

1) Weak-form efficiency: The current prices of assets encompass all past information, rendering technical analysis based on price history incapable of predicting future price movements.

2) Semi-strong-form efficiency: The current asset prices encapsulate all public information, comprising both historical and present data, as well as any information publicly available. This implies that neither fundamental nor technical analysis should result in extraordinary profits.

3) Strong-form efficiency: Current asset prices incorporate all information, whether public or private. According to this level of efficiency, even access to privileged information should not lead to abnormal profits.

Despite the widespread research and discussions surrounding the Efficient Market Hypothesis, it is not impervious to critique. Some dissenting voices argue that market efficiency is not constant due to factors such as irrational investor behavior, information asymmetry, transaction costs, and more. Instances of financial bubbles and historical stock market crashes are often invoked to challenge the notion of perfect market efficiency.

Nevertheless, the Efficient Market Hypothesis remains a vital theoretical framework for comprehending the functioning of financial markets, albeit triggering ongoing debates and controversies within the financial realm.

The primary method for empirically assessing the Efficient Market Hypothesis (*EMH*) in its weak form involves examining the behavior of market prices through the lens of random-walk processes. The random walk model posits that prices in financial markets follow a random-walk pattern, whether with or without a drift. Consequently, attempting to forecast the future value of assets based on identifying trends or patterns in price changes within a market is deemed ineffective.

The weak form of the Efficient Market Hypothesis asserts that there should be no discernible correlation in price movements over time, a proposition subject to empirical testing. One statistical approach involves investigating auto (or serial) correlation, where the movement of a stock's price in one period is scrutinized concerning its movements in a preceding period. Specifically, serial autocorrelation examines the correlation between a time series and its own values at various lags. A negative outcome in serial autocorrelation suggests mean reversion, supporting the null hypothesis, while positive coefficients lead to the rejection of the null hypothesis.

In order to assess the random walk hypothesis (*RWH*), we conducted a series of tests in a paper Soukrati (2024)to evaluate the weak form informational efficiency of the Moroccan stock market through its *MASI* index, namely unit root tests, including the Augmented Dickey-Fuller test, the Phillips-Perron test, and the Kwiatkowski, Phillips, Schmidt, and Shin test; normality tests such as the Jarque-Bera test, the Kolmogorov-Smirnov test, the Lilliefors test, the Cramer-von Mises test, the Anderson-Darling test, and the quantile-quantile plot-based test; the Ljung-Box serial correlation test; the variance ratio test; and the Runs test.The results we obtained from all applied tests indicate that the Moroccan stock market is inefficient in its weak form.

All statistical techniques in the literature testing the random walk hypothesis are characterized by the fact that they do not account for long memory. Recent developments in statistical literature have delved into the exploration of long memory models that extend beyond the realm of random walks and unit roots in univariate time series representations. $ARFIMA$ (p, d, q) processes have emerged as adept models for capturing long-run persistence. These processes, introduced initially by Granger and Joyeux (1980), extend the scope of Box-Jenkins models by permitting the order d of integration to take fractional values.

In this paper we test the weak form of the informational efficiency hypothesis of the Moroccan stock exchange by analyzing the properties of long memory through $ARFIMA$ (Autoregressive Fractionally Integrated Moving Average) models.

The rest of this study is arranged as follows: Section two covers the literature review, section three details the study's data and methodology, section four presents and discusses the results, and section five concludes.

2. Literature Review

Numerous research works have explored informational efficiency by analyzing the long-term memory of stock market indices through the utilization of $ARFIMA$ models.

Kang and Yoon (2006) explored the existence of long memory in the returns of indices from four Asian stock markets, namely Japan, South Korea, Hong Kong, and Singapore, using the $ARFIMA$ model. Additionally, they assessed the presence of long-term memory in the conditional variances of these indices using the $FIEGARCH$ model. The results of the $ARFIMA$ model revealed no evidence of long memory in the returns of the four indices, while the results of the $FIEGARCH$ model detected long memory in the volatilities of the four indices.

In a comparable study, Nazarian et al. (2014) utilized the $ARFIMA$ model to capture long-term memory in the returns of the Tehran Stock Exchange (TSE) index. Simultaneously, they employed the $FIGARCH$ model to identify long memory in the conditional variance (volatility) of the TSE index. The results of the $ARFIMA$ model suggested a lack of long memory in the return series of the TSE index, while the results from the $FIGARCH$ model revealed evidence of long memory in the conditional variance of this series.

Similarly, Turkyilmaz and Balibey (2014) investigated the weak form efficiency of the Karachi stock market in Pakistan spanning from 2010 to 2013, employing the $ARFIMA\text{-}FIGARCH$ model. The study's findings revealed that the $ARFIMA$ component of the model did not provide evidence supporting long-term memory behavior in the returns of the Karachi market. However, the $FIGARCH$ component of the model indicated that the volatility of the market's returns exhibits long memory.

In a parallel study, Dodi et al. (2015) applied $ARFIMA$ to model opening price of Kedaung Indah Can Tbk Stock of Indonesian Stock Exchange from May 2nd 2005 until March 26th 2012. They found that the best suited model is $ARFIMA(5{,}0.452{,}4)$ where for short time forcasting is shown very close to actual stock price with small standard error.

Likewise, Mahboob et al. (2017) conducted a study to analyze the presence of long memory in the daily returns and volatilities of the Dhaka Stock Exchange index in Bangladesh. The study spanned from December 15, 2003, to July 31, 2013, and employed both $ARFIMA\text{-}FIGARCH$ and $ARFIMA\text{-}FIPARCH$ models. The test results revealed the existence of long memory in both returns and volatilities within the Dhaka stock market.

In separate research, Houfi (2019) conducted a study to assess the weak form of informational efficiency in the Tunisian stock exchange. The author scrutinized the presence of long memory in the series of daily returns and volatility of the Tunisian stock index, utilizing the $ARIMA\text{-}FIGARCH$ model. The empirical study encompassed a sample period from January 2, 1998, to March 16, 2018. The findings indicated the existence of long memory in both the returns and volatility of the Tunisian stock market.

In a related study, Fakhriyana et al. (2019) applied $ARIMA$-$GARCH$ and $ARFIMA$-$EGARCH$ models to evaluate the long memory and high volatility of the Jakarta Composite Index (*JCI*), encompassing all listed stocks on the Indonesian stock exchange, during the timeframe from 2007 to 2018. The conclusion in this study is that *JCI* exhibits long memory characteristics and demonstrates an asymmetric impact.

In a study conducted by Lamouchi in 2020, the *ARFIMA* model was employed on the Tadawul index of the Saudi stock market to capture the long memory of daily returns spanning from 1998 to 2020. The findings revealed a presence of long-term memory in the Saudi stock market, challenging the Efficient Market Hypothesis (*EMH*).

In an analogous research, Falloul (2020) conducted a comparable study to assess the weak form of the efficiency hypothesis in the Moroccan stock market. The *ARFIMA* model was applied to analyze the daily returns of the *MASI* index. The findings indicated the presence of long memory in the Moroccan stock market, thereby rejecting the efficiency hypothesis for this market.

A similar study was conducted by Alfred and Sivarajasingham (2020) to examine the efficiency hypothesis of the daily return series of the Sri Lankan stock index spanning from January 2, 1985, to September 28, 2018. They utilized the *ARFIMA* model to identify long memory in the return series and applied the *FIGARCH* model to capture long memory in the conditional volatility of the Sri Lankan index. The findings revealed that the return series lacks long memory, whereas the volatility series exhibits prolonged memory.

In a parallel study, Ziky and Ouali (2021) examined the weak form efficiency of the Moroccan stock market by employing the *ARFIMA* model to identify long memory in the daily return series of the *MASI* index spanning from 1992 to 2016. The outcomes suggested that the Moroccan stock market displays long memory, leading to the conclusion that it can be deemed inefficient.

In a comparable context, Bouchareb et al. (2021) utilized the *ARFIMA-FIGARCH* model to identify long memory in both returns and volatilities across four Mediterranean stock markets: Morocco, Turkey, Spain, and France. The study spanned the period from 2000 to 2020. The findings presented compelling evidence of long memory in both returns and volatilities for the Moroccan and French stock markets. However, the evidence was limited to volatility alone for the Spanish and Turkish markets, leading to the rejection of the efficiency hypothesis for these markets.

Odonkor et al. (2022) investigated the presence of long memory in the daily returns and volatilities of seven stocks listed on the Ghana Stock Exchange, employing the *ARFIMA-FIGARCH* model. The study revealed that all seven stocks demonstrated long memory in both returns and volatility, which contradicts the efficiency hypothesis of the Ghanaian stock market.

Acim et al. (2022) study the characteristic of long memory for the Sukuk series of the Islamic financial market of Malaysia using *ARFIMA* model. They use government indexes of several maturities while comparing them with their counterparts in each maturity, over the periods from 2007 to 2017. They found the presence of the long memory for conventional bonds of short and medium maturity while they captured it for Sukuk of long maturity.

3. Data and Methodology

3.1 Data

The dataset for this study comprises the daily closing prices of the Casablanca Stock Exchange index (*MASI*), spanning from March 1, 2002, to January 23, 2024, with a total of 5502 observations. The *MASI*, representing the Moroccan All Shares index, is a weighted index that encompasses all equity-type securities listed on the Casablanca Stock Exchange. Therefore, it serves as a comprehensive indicator, offering an effective means to monitor the evolution of the complete set of listed securities.

Thereafter, the *MASI* index prices were transformed into geometric returns:

$$r_t = ln\left(\frac{P_t}{P_{t-1}}\right) = ln(P_t) - ln(P_{t-1}) \tag{1}$$

Where P_t represents the index price, and ln corresponds to the natural logarithm.

The dataset was acquired through downloading from the www.investing.com website.

3.2 Methodology

In this section, we describe the *ARFIMA* model designed to capture the long memory of returns.

- ***Auto Regressive Mobile Average model : $ARMA(\bar{p},\bar{q})$***

Let (X_t) be a stationary stochastic process. We say that (X_t) is an $ARMA(\bar{p},\bar{q})$ process of orders $\bar{p}$ and $\bar{q}$ if there exist lag polynomials $\phi(L)$ of order $\bar{p}$ and $\psi(L)$ of order $\bar{q}$ whose roots are all outside the unit circle, and a white noise (ε_t) such that:

$$\phi(L)X_t = c + \psi(L)\varepsilon_t \tag{2}$$

where L is the lag operator, and ϕ and ψ are lag polynomials defined by:

$$\phi(L) = 1 - \sum_{i=1}^{\bar{p}} \phi_i L^i \quad \text{avec } \phi_p \neq 0 \qquad \psi(L) = 1 + \sum_{j=1}^{\bar{q}} \psi_j L^j \quad \text{avec } \psi_q \neq 0 \tag{3}$$

The $ARMA(\bar{p},\bar{q})$ process can also be expressed as:

$$X_t = c + \sum_{i=1}^{\bar{p}} \phi_i X_{t-i} + \sum_{j=0}^{\bar{q}} \psi_j \varepsilon_{t-j} + \varepsilon_t \tag{4}$$

- ***AutoRegressive Integrated Moving Average model : $ARIMA(\bar{p},d,\bar{q})$***

Let (X_t) be a non-stationary integrated stochastic process of order $d \in \mathbb{N}^*$. We say that (X_t) is an $ARIMA(\bar{p},d,\bar{q})$ process of orders $\bar{p}$, d and $\bar{q}$ if there exist lag polynomials $\phi(L)$ of order $\bar{p}$ and $\psi(L)$ of order $\bar{q}$, with roots all outside the unit circle, and a white noise (ε_t) such that:

$$\phi(L)(1-L)^d X_t = c + \psi(L)\varepsilon_t \tag{5}$$

where L is the lag operator, and ϕ and ψ are defined as previously.

- ***AutoRegressive Fractionally Integrated Mobile Average model : $ARFIMA(\bar{p},d,\bar{q})$***

We say that (X_t) is an $ARFIMA(\bar{p},d,\bar{q})$ process of orders $\bar{p}, d \in \mathbb{Q}$ and$\bar{q}$ if there exist lag polynomials $\phi(L)$ of order $\bar{p}$, $\psi(L)$ of order $\bar{q}$ with all roots outside the unit circle, and white noise(ε_t) such that:

$$\phi(L)(1-L)^d X_t = c + \psi(L)\varepsilon_t \tag{6}$$

where L is the lag operator, and ϕ and ψ are defined as previously:

The filter $(1-L)^d$ can be expressed in the form:

$$(1-L)^d = \sum_{j=0}^{\infty} (-1)^j \binom{d}{j} L^j = \sum_{j=0}^{\infty} \frac{\Gamma(j-d)}{\Gamma(-d)\Gamma(j+1)} L^j \tag{7}$$

where $\binom{d}{j}$ is the binomial coefficient, and $\Gamma(.)$is the gamma function.

- ***Properties of an $ARFIMA(\bar{p},d,\bar{q})$ process:***

Let (X_t)be an $ARFIMA(\bar{p},d,\bar{q})$ process. Then:

- If $-1/2 < d < 1/2$, the$ARFIMA(\bar{p},d,\bar{q})$ process is stationary with an autocorrelation function $\rho(k)$ that decreases hyperbolically:

$$\rho(k) \sim C.k^{2d-1}$$

- If $0 < d < 1/2$, and if all the roots of $\phi(L) = 0$ are outside the unit circle, then the $ARFIMA(\bar{p},d,\bar{q})$process is stationary with long memory. Autocorrelations are positive and decrease hyperbolically towards zero as the lag increases.
- If $-1/2 < d < 0$, the $ARFIMA(\bar{p},d,\bar{q})$process is stationary and anti-persistent (intermediate persistence). Autocorrelations decrease hyperbolically towards zero, and the spectral density is dominated by high-frequency components (it tends to zero as frequency tends to zero).
- If $d \geq 1/2$, then the $ARFIMA(\bar{p},d,\bar{q})$ process is non-stationary.
- If $d = 0$, the $ARFIMA(\bar{p},d,\bar{q})$ process reduces to the standard $ARFIMA(\bar{p},\bar{q})$process with short memory (where the effect of a random shock fades exponentially over time).
- If $d = 1$, we obtain the $ARIMA(\bar{p},1,\bar{q})$ process.

4. Results and Discussion

Before investigating the presence of long memory in the financial time series of the Moroccan stock index, we perform standard statistical tests. These include examining histogram and descriptive statistics, conducting normality tests, and evaluating the stationarity of the return's series.

To begin, we visually depict the daily geometric returns series of the *MASI* index from 03/01/2002 to 22/01/2024.

Figure 1 below describe the plot of daily geometric returns of *MASI* Index.

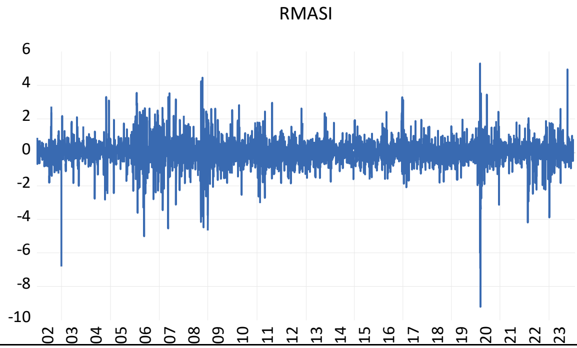

Figure 1: Geometric Returns of *MASI* Index

4.1 Histograms, descriptive statistics and standard statistical tests

The figure 2 below shows the histogram and descriptive statistics of the geometric returns of the MASI index during the period from 03/01/2002 to 22/01/2024.

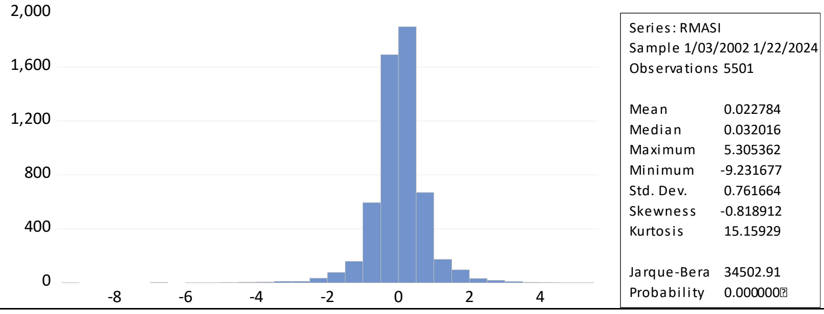

Figure 2: Histogram and Descriptive Statistics of the geometric returns series of the *MASI* Index

It is notable that both the average and median returns are positive. The skewness coefficient for returns deviates from zero, demonstrating a negative skewness. Additionally, the kurtosis coefficient surpasses three, signifying a higher than normal level of peakedness in the distribution. The combination of negative skewness and increased kurtosis suggests a significant departure from normality in the unconditional distribution of returns.

Furthermore, the Jarque-Bera statistic, applied to test the hypothesis of a normal distribution of returns, rejects this hypothesis at a significance level of 1%. Consequently, we can dismiss the null hypothesis of normality in the level series for the *MASI* index returns.

Below, we applied the Augmented Dickey Fuller test to the series of geometric returns of the *MASI* index in its three forms: ADF without a constant and trend, *ADF* with a constant, and *ADF* with a constant and trend. The results are provided in the tables 1, 2 and 3.

Table 1: Result of the *ADF* test without a constant and without trend

Null Hypothesis: RMASI has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=32)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-57.62284	0.0001
Test critical values:	1% level	-2.565383	
	5% level	-1.940882	
	10% level	-1.616661	

*MacKinnon (1996) one-sided p-values.

Table 2: Result of the ADF test with a constant

Null Hypothesis: RMASI has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=32)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-57.65826	0.0001
Test critical values:	1% level	-3.431358	
	5% level	-2.861870	
	10% level	-2.566988	

*MacKinnon (1996) one-sided p-values.

Table 3: Result of the *ADF* test with a constant and with a trend

Null Hypothesis: RMASI has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=32)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-57.67746	0.0000
Test critical values:	1% level	-3.959706	
	5% level	-3.410621	
	10% level	-3.127089	

*MacKinnon (1996) one-sided p-values.

The results of the Augmented Dickey-Fuller (*ADF*) unit root tests reveal that the t-statistic for all three tests is considerably lower than the critical values at the 1%, 5%, and 10% significance levels. This suggests that the series of geometric returns for the *MASI* index exhibits stationarity.

We also employ the Quantile-Quantile plot (QQ-Plot) test to assess the conformity of the geometric returns of *MASI* to a normal distribution. A perfectly fitting distribution would result in the QQ-Plot aligning along a straight line at 45 degrees, indicating similarity between the empirical and theoretical (normal) distributions. Figure 3 below depicts the QQ-Plot illustrating the empirical distribution of the geometric returns of *MASI*.

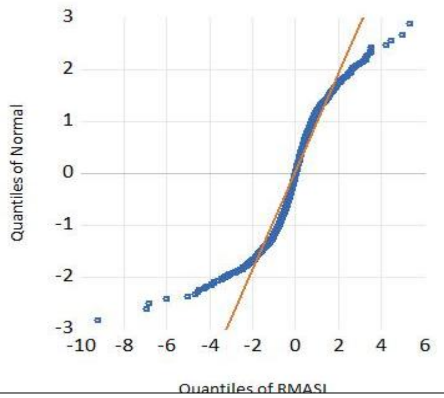

Figure 3: QQ-Plot of the geometric return of MASI

Upon inspecting Figure 3, it becomes evident that the geometric returns of *MASI*, as depicted in the empirical distribution, deviate from normality. The tails of the distribution appear thicker than those of a normal distribution. The QQ-Plot deviates from a straight line, displaying an S-shape.

4.2 Assessing long memory of the geometric return's series of *MASI*

In this section, we tested the efficiency of the Moroccan stock market by analyzing the long memory property in conditional mean.

The *ARFIMA* model parameters were computed. Multiple *ARFIMA* (p, d, q) models were estimated for $1 \leq p \leq 3$ and $1 \leq q \leq 3$, and only those models with all significant parameters were chosen. The results are presented in the tables below.

Table 4 : *ARFIMA* $(1, d, 0)$ model

Parameter	Coefficient	Std. Error	t-value	t-prob
d-ARFIMA	**0.100215**	0.02051	4.89	0.000
AR-1	0.144284	0.02366	6.10	0.000
log-likelihood	0.302110749			
no. of observations	5501			
no. of parameters	3			
AIC.T	5.3957785			
AIC	0.000980872296			
mean(RMASI)	0.0227838			
var(RMASI)	0.580027			
sigma	0.739256			
sigma^2	0.5465			

(log-likelihood and standard errors based on NLS function)

Table 5 : *ARFIMA* $(0, d, 1)$ model

Parameter	Coefficient	Std. Error	t-value	t-prob
d-ARFIMA	**0.100215**	0.01648	6.08	0.000
MA-1	0.151626	0.01878	8.08	0.000
log-likelihood	0.302147491			
no. of observations	5501			
no. of parameters	3			
AIC.T	5.39570502			
AIC	0.000980858938			
mean(RMASI)	0.0227838			
var(RMASI)	0.580027			
sigma	0.739229			
sigma^2	0.54646			

(log-likelihood and standard errors based on NLS function)

Table 6 : $ARFIMA\ (2,d,1)$ model

Parameter	Coefficient	Std. Error	t-value	t-prob
d-ARFIMA	**0.100215**	0.05892	1.70	0.089
AR-1	2.80895	0.1272	22.1	0.000
AR-2	-0.405708	0.1771	-2.29	0.022
MA-1	-0.315186	0.1146	-2.75	0.006
log-likelihood	-0.677854032			
no. of observations	5501			
no. of parameters	5			
AIC.T	11.3557081			
AIC	0.00206429887			
mean(RMASI)	0.0227838			
var(RMASI)	0.580027			
sigma	1.96965			
sigma^2	3.87951			

(log-likelihood and standard errors based on NLS function)

Table 7 : $ARFIMA\ (2,d,2)$ model

Parameter	Coefficient	Std. Error	t-value	t-prob
d-ARFIMA	**0.100215**	0.02119	4.73	0.000
AR-1	0.740498	0.1789	4.14	0.000
AR-2	-0.411761	0.07261	-5.67	0.000
MA-1	-0.595015	0.1814	-3.28	0.001
MA-2	0.308425	0.09224	3.34	0.001
log-likelihood	0.303910562			
no. of observations	5501			
no. of parameters	6			
AIC.T	11.3921789			
AIC	0.00207092872			
mean(RMASI)	0.0227838			
var(RMASI)	0.580027			
sigma	0.737927			
sigma^2	0.544536			

(log-likelihood and standard errors based on NLS function)

Table 8 : $ARFIMA\ (1,d,3)$ model

Parameter	Coefficient	Std. Error	t-value	t-prob
d-ARFIMA	**0.100215**	0.05800	1.73	0.084
AR-1	0.737587	0.08781	8.40	0.000
MA-1	-0.583500	0.1216	-4.80	0.000
MA-2	-0.0941258	0.01945	-4.84	0.000
MA-3	-0.0600335	0.01640	-3.66	0.000
log-likelihood	0.304491319			
no. of observations	5501			
no. of parameters	6			
AIC.T	11.3910174			
AIC	0.00207071757			
mean(RMASI)	0.0227838			
var(RMASI)	0.580027			
sigma	0.737498			
sigma^2	0.543904			

(log-likelihood and standard errors based on NLS function)

The long memory parameters *d-ARFIMA*of the $ARFIMA\ (1, d, 0)$, $ARFIMA\ (0, d, 1)$ and $ARFIMA\ (2, d, 2)$ models show statistical significance at a 1% significance level. On the other hand, the long memory parameters *d-ARFIMA* of the $ARFIMA\ (2, d, 1)$ and $ARFIMA\ (1, d, 3)$models exhibit statistical significance at a 10% significance level.

The other parameters, of all models, are statistically significant at the 1% level, except for the parameter corresponding to $MA\,(1)$ in the $ARFIMA\ (2, d, 1)$ model, which is statistically significant at 5%.

The $d - ARFIMA$ value for all models is determined by:

$$d - ARFIMA = 0.100215$$

Since$0 < d - ARFIMA < 1/2$, we infer that the five$ARFIMA\ (p, d, q)$processare stationary with long memory. Autocorrelations are positive and decrease hyperbolically towards zero as the lag increases.

The table 9 below compare the values of the *AIC*information criteria for the five models.

Table 9 : Comparison of Akaike Information Criterion for five models, $d = 0.100215$

Model	Akaike Information Criterion AIC
$ARFIMA\ (1, d, 0)$	0.000980872296
$ARFIMA\ (0, d, 1)$	0.000980858938
$ARFIMA\ (2, d, 1)$	0.00206429887
$ARFIMA\ (2, d, 2)$	0.00207092872
$ARFIMA\ (1, d, 3)$	0.00207071757

As we can see in this table, it is the model $ARFIMA\ (0, d, 1)$that minimizes the AIC information criteriawith the value 0.000980858938. The $ARFIMA\ (1, d, 0)$ model exhibits an *AIC* criterion almost equal to that of the $ARFIMA\ (0, d, 1)$model with the value 0.000980858938.

As evident from the table, the $ARFIMA\ (0, d, 1)$ model minimizes the *AIC* information criteria, presenting a value of 0.000980858938. The $ARFIMA\ (1, d, 0)$model closely follows, displaying an *AIC* criterion almost identical to that of the $ARFIMA\ (0, d, 1)$model, with the value 0.000980858938.

In summary, we can assert that all five estimated models effectively capture the long memory property in the returns of the *MASI* index, contradicting the market efficiency hypothesis.

5. Conclusion

This research explored the weak form efficiency of the Moroccan stock market by examining the presence of the long memory phenomenon. To address this, we applied the *ARFIMA* model to the daily geometric returns of the *MASI* index spanning from March 1, 2002, to January 22, 2024.

Prior to delving into the investigation of long memory in the financial time series of the Moroccan stock index, we conducted standard statistical tests. The Jarque-Bera test and the Quantile-Quantile plot test were employed, leading to the rejection of normality in the geometric return series of the *MASI* index. Additionally, the Augmented Dickey-Fuller test confirmed the stationarity of the geometric return series of the *MASI* index.

Afterward, we have estimated several $ARFIMA\ (p, d, q)$ models for $1 \le p \le 3$ and $1 \le q \le 3$ and only models whose the parameters are all significant were selected. We have retained finally five models, namely $ARFIMA\ (1, d, 0)$, $ARFIMA\ (0, d, 1)$, $ARFIMA\ (2, d, 1)$ and $ARFIMA\ (1, d, 3)$. The long memory parameter, represented as d-ARFIMA, is consistent across all five models, with a value of 0.100215.Since $0 < d - ARFIMA < 1/2$, we infer that the $ARFIMA\ (p, d, q)$ processes for all five models are stationary with long memory. The autocorrelations exhibit a positive trend, decreasing hyperbolically towards zero as the lag increases.

Our results lead to the conclusion that the Moroccan stock market exhibits informational weak-form inefficiency. Consequently, these markets are often deemed attractive for investors who can develop strategies based on the over or undervaluation of stock prices, potentially leading to abnormal returns.

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