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Reliability Analysis of a Parallel Unit System with Two Cold Standby Units

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Abstract: The present study deals with the reliability analysis of a parallel unit system with two cold standby units. In the beginning, there is one main unit and two cold standby units. The system remains in operable state until its complete failure and whenever system comes across any halt, both cold standby units start functioning together in order to keep the system operating. There is single repairman available for repair of main as well as cold standby unit. The reliability and profit analysis has been done for the present model. Various measures of system effectiveness such as MTSF and Profit are obtained using semi Markov process and Regenerative point technique.
Keywords: Standby systems; semi Markov process; Regenerative point technique.

Introduction

Reliability is the ability of a system to perform its intended function under given circumstances and within specified time interval. The standby systems holds a great importance in the field of reliability engineering. Various reliability models have been used by researchers under different circumstances [1-7]. The standby unit helps the system to sustain in presence of main unit's failure. Therefore, our study deals with such situation.

The complete system consists of one main unit and two cold standby units. In the beginning, there is one main unit which is in operative state and two cold standby units which are kept spare. The cold standby units come into operation mode whenever there is any fault in main unit and it stops functioning. Both cold standby units starts functioning together as the capacity of both cold standby units to bring out smooth function of complete system is equivalent to that of one main unit. There is only one repairman available to do the job of repair for main as well as cold standby units. Various measures of system effectiveness such as MTSF and Profit are obtained using semi Markov process and Regenerative point technique. The graphical interpretation has also been done for the present study.

Assumptions

1. There are no different repairmen for main unit and cold standby units i.e. there is single repairman facility available.
2. Working of both cold standby systems will keep the system operating.
3. At an instance, only one unit of cold standby system can fail, i.e. Failure in both cold standby units cannot occur simultaneously.
4. Only one failure can occur at a time.

Notations

λ	Rate of occurrence of failure in main unit
λ_1/ λ_2	Rate of occurrence of failure in I^{st} / II^{nd} cold standby unit
g(t)/ G(t)	pdf/ cdf of times to repair the main unit at failed state
$g_1(t)$/ $G_1(t)$	pdf/ cdf of times to repair the I^{st} cold standby unit at failed state
$g_2(t)$/ $G_2(t)$	pdf/ cdf of times to repair the II^{nd} cold standby unit at failed state
O_I/ O_{II}/ O_{III}	I^{st}/ II^{nd}/ III^{rd} unit under operation
S_{II}/ S_{III}	II^{nd}/ III^{rd} unit under cold standby state
F_{rI}/F_{wrI}	I^{st} unit under repair/ waiting for repair
F_{rII}/F_{wrII}	II^{nd} unit under repair/ waiting for repair

F_{rIII}/F_{wrII}	III[rd] unit under repair/ waiting for repair
F_{RI}	I[st] unit under repair continuing from the previous state
F_{RII}	II[nd] unit under repair continuing from the previous state
F_{RIII}	III[rd] unit under repair continuing from the previous state

Transition Probabilities and Mean Sojourn Times

The possible states of the system with current status are provided Table No. I and the transition rates are given in Table No. II. The epochs of entry into states 0, 1, 4 and 5 are regenerative points and thus these are regenerative states. The states 2 and 3 are failed states.

State No.	Status
S_0	O_I , S_{II} , S_{III}
S_1	F_{rI} , O_{II} , O_{III}
S_2	F_{RI} , F_{wrII} , S_{III}
S_3	F_{RI} , S_{II} , F_{wrIII}
S_4	O_I , F_{rII} , S_{III}
S_5	O_I , S_{II} , F_{rIII}

Table No. I: Possible States with Status

S.No.	From State	To State	Rate
1	S_0	S_1	λ
2	S_1	S_0	g(t)
3	S_1	S_2	λ_1
4	S_1	S_3	λ_2
5	S_2	S_4	g(t)
6	S_3	S_5	g(t)
7	S_4	S_0	$g_1(t)$
8	S_5	S_0	$g_2(t)$

Table No. II: Transition Rate

Transition Probabilities:

The transition probabilities are given by:

$$dQ_{01}(t)=\lambda e^{-\lambda t}dt \qquad dQ_{10}(t)=g(t)e^{-(\lambda_1+\lambda_2)t}dt$$

$$dQ_{12}(t)=\lambda_1 e^{-(\lambda_1+\lambda_2)t}\overline{G}(t) \qquad dQ_{13}(t)=\lambda_2 e^{-(\lambda_1+\lambda_2)t}\overline{G}(t)$$

$$dQ_{24}(t)=g(t)dt \qquad dQ_{35}(t)=g(t)dt$$

$$dQ_{40}(t)=g_1(t)dt \qquad dQ_{50}(t)=g_2(t)dt$$

$$dQ_{14}^{(2)}(t)=(\lambda_1 e^{-(\lambda_1+\lambda_2)t}\copyright 1)g(t)dt \qquad dQ_{15}^{(3)}(t)=(\lambda_2 e^{-(\lambda_1+\lambda_2)t}\copyright 1)g(t)dt$$

The non-zero elements p_{ij}, are obtained as under:

$p_{01} = 1$ $\qquad p_{10} = g^*(\lambda_1 + \lambda_2)$

$p_{12} = \frac{\lambda_1[1 - g^*(\lambda_1 + \lambda_2)]}{\lambda_1 + \lambda_2} = p_{14}^{(2)}$ $\qquad p_{40} = g_1^*(0)$

$p_{13} = \frac{\lambda_2[1 - g^*(\lambda_1 + \lambda_2)]}{\lambda_1 + \lambda_2} = p_{15}^{(3)}$ $\qquad p_{50} = g_2^*(0)$

$p_{24} = g^*(0) = p_{35}$

By these transition probabilities, it can be verified that

$p_{01} = 1 \qquad p_{10} + p_{14}^{(2)} + p_{15}^{(3)} = 1 \qquad p_{10} + p_{12} + p_{13} = 1 \qquad p_{24} = 1$

$p_{35} = 1 \qquad p_{40} = 1 \qquad p_{50} = 1$

The unconditional mean time taken by the system to transit for any regenerative state j, when it is counted from epoch of entrance into that state i, is mathematically stated as –

$$m_{ij} = \int_0^\infty t\,dQ_{ij}(t) = -q_{ij}^{*'}(0), Thus -$$

$m_{01} = \frac{1}{\lambda} \qquad m_{10} + m_{12} + m_{13} = \mu_1$

$m_{24} = k_1 = m_{35} \qquad m_{40} = k_2 \qquad m_{50} = k_3$

where,

$k_1 = \int_0^\infty \overline{G}(t)dt \qquad k_2 = \int_0^\infty \overline{G_1}(t)dt \qquad k_3 = \int_0^\infty \overline{G_2}(t)dt$

The mean sojourn time in the regenerative state i (μ_i) is defined as the time of stay in that state before transition to any other state, then we have -

$\mu_0 = \frac{1}{\lambda} \qquad \mu_1 = \frac{1 - g^*(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2}$

$\mu_2 = -g^*(0) = \mu_3 \qquad \mu_4 = -g_1^*(0)$

$\mu_5 = -g_2^*(0)$

Mean Time to System Failure

The expressions for the mean time to system failure (MTSF) are obtained on taking the failed states of the system as absorbing states. By probabilistic arguments, we obtain the following recursive relations for $\phi_i(t)$, c.d.f. of the first passage time from regenerative state i to failed state :

$$\phi_0(t) = Q_{01}(t)(s)\phi_1(t)$$
$$\phi_1(t) = Q_{10}(t)(s)\phi_0(t) + Q_{12}(t) + Q_{13}(t)$$
$$\phi_4(t) = Q_{40}(t)(s)\phi_0(t)$$
$$\phi_5(t) = Q_{50}(t)(s)\phi_0(t)$$

Taking Laplace Stieltjes Transformation of these equations and solving for $\phi_0^{**}(s)$, we obtain

$$\phi_0^{**}(s) = \frac{N(s)}{D(s)}$$

The mean time to system failure when the system starts from the state 0, is

$$T_0 = \lim_{s\to 0} R^*(s) = \lim_{s\to 0} \frac{1-\phi_0^{**}(s)}{s} = \frac{N}{D}$$

Where $R^*(s)$ is the Laplace Transformation of the Reliability R(t). The Reliability R(t) of the system at time 't' can be obtained taking inverse Laplace transform of $R^*(s)$ Using L 'Hospital rule and putting the value of $\phi_0^{**}(s)$ we have

$$N = (\mu_0 + \mu_1)$$
$$D = 1 - p_{10}$$

Expected Up-Time of the System

Using the arguments of the theory of regenerative processes, the availability $AF_i(t)$, the probability that the system is up at instant 't' with full capacity given that it entered regenerative state 'i' at time t = 0, is seen to satisfy the following recursive relations.

$$AF_0(t) = M_0(t) + q_{01}(t)© AF_1(t)$$
$$AF_1(t) = M_1(t) + q_{10}(t)© AF_0(t) + q_{14}^{(2)}(t)© AF_4(t) + q_{15}^{(3)}(t)© AF_5(t)$$
$$AF_4(t) = M_4(t) + q_{40}(t)© AF_0(t)$$
$$AF_5(t) = M_5(t) + q_{50}(t)© AF_0(t)$$

Taking Laplace transform of the above equations and solving for $AF_0^{**}(s)$, we have

$$AF_0^{**}(s) = \frac{N_1(s)}{D_1(s)}$$

The steady state availability of the system is given by

$$AF_0 = \lim_{s\to 0} (sAF_0^*(s)) = \frac{N_1}{D_1}$$

Where

$$M_0(t) = e^{-\lambda t}dt \qquad M_1(t) = e^{-(\lambda_1+\lambda_2)}\overline{G}(t)$$
$$M_4(t) = \overline{G_1}(t) \qquad M_5(t) = \overline{G_2}(t)$$
$$N_1 = \mu_0 + \mu_1 + k_2 p_{14}^{(2)} + k_3 p_{15}^{(3)}$$
$$D_1 = \mu_0 + \mu_1 + k_2 p_{14}^{(2)} + k_3 p_{15}^{(3)}$$

Busy Period of a Repairman

Using the probabilistic arguments for regenerative process, the following recursive relation for $B_i(t)$ are obtained.

$$B_0(t) = q_{01}(t)©B_1(t)$$
$$B_1(t) = q_{10}(t)©B_0(t) + q_{14}^{(2)}(t)©B_4(t) + q_{15}^{(3)}(t)©B_5(t)$$
$$B_4(t) = W_4(t) + q_{40}(t)©B_0(t)$$
$$B_5(t) = W_5(t) + q_{50}(t)©B_0(t)$$

The steady state busy period of the system is given by:

$$B_R = \frac{N_2}{D_1}$$
$$N_2 = k_2 p_{14}^{(2)} + k_3 p_{15}^{(3)}$$

And D_1 is already specified above.

Expected No of Visits of Repairman

Using the probabilistic arguments for regenerative process, the following recursive relation for $V_i(t)$ are obtained.

$$V_0(t) = Q_{01}(t)(s)[1+V_1(t)]$$
$$V_1(t) = Q_{10}(t)(s)V_0(t) + Q_{14}^{(2)}(t)(s)V_4(t) + Q_{15}^{(3)}(t)(s)V_5(t)$$
$$V_4(t) = Q_{40}(t)(s)V_0(t)$$
$$V_5(t) = Q_{50}(t)(s)V_0(t)$$

The steady state expected no. of visits of the repairman is given by:

$$V_R = \frac{N_3}{D_1}$$
$$N_3 = N_3(0) = 1$$

And D_1 is already specified above.

Profit Analysis

The expected profit incurred of the system is -

$$P = C_0 AF_0 - C_1 B_R - C_2 V_R$$

C_0 = Revenue per unit up time of the system
C_1 = Cost per unit up time for which the repairman is busy in repair
C_2 = Cost per visit of the repairman

Graphical Interpretation and Conclusion

For graphical analysis following particular cases are considered:

$g(t)=\beta e^{-\beta t}$ $\quad g_1(t)=\beta_1 e^{-\beta_1 t}$ $\quad g_2(t)=\beta_2 e^{-\beta_2 t}$

$p_{01}=1$ $\quad p_{10}=\dfrac{\beta}{\lambda_1+\lambda_2+\beta}$

$p_{12}=\dfrac{\lambda_1}{\lambda_1+\lambda_2+\beta}=p_{14}^{(2)}$ $\quad p_{13}=\dfrac{\lambda_2}{\lambda_1+\lambda_2+\beta}=p_{15}^{(3)}$

$p_{40}=1$ $\quad p_{50}=1$ $\quad p_{24}=1=p_{35}$

$\mu_0=\dfrac{1}{\lambda}$ $\quad \mu_1=\dfrac{1}{\lambda_1+\lambda_2+\beta}$

$\mu_2=\dfrac{1}{\beta}=\mu_3$ $\quad \mu_4=\dfrac{1}{\beta_1}$

$\mu_5=\dfrac{1}{\beta_2}$

	T_0 (in Hrs.)		
	λ_1 = 0.000015	λ_1 = 0.000020	λ_1 = 0.000025
0.000024	16966908	12817058	10325565
0.000026	15638231	11812254	9515258
0.000028	14500408	10951813	8821391
0.00003	13515213	10206819	8220641.5
0.000032	12653979	9555584	7695515.5
0.000034	11894784	8981530	7232640.5
0.000036	11220585	8471768	6821618.5
0.000038	10617931	8016115.5	6454240
0.00004	10076058	7606437	6123937.5
0.000042	9586259	7236139.5	5825401

Table No. III

The behaviour of MTSF w.r.t. V/s rate of failure of main unit (λ) for different values of rate of failure of Ist standby unit (λ_1) has been given in Table No. III. From the table, it can be interpreted that MTSF gets decreased with the increase in the values of the failure of main unit (λ). It is also been interpreted that with the increase in rate of failure of Ist standby unit (λ_1), the MTSF decreases.

	PROFIT (in INR)		
C_0	λ = 0.000032	λ = 0.0032	λ = 0.32
200	-166.168594	-20264.27734	-77926.46875
15200	14833.6543	-5279.737793	-65993.14453
30200	29833.47852	9704.800781	-48059.82031
45200	44833.30078	24689.33984	-33126.5
60200	59833.12109	39673.87891	-18193.17969
75200	74832.9375	54658.41406	-3259.859863
90200	89832.76563	69642.95313	11673.46777
105200	104832.5859	84627.49219	26606.78906

120200	119832.4063	99612.03125	41540.10938
135200	134832.2344	114596.5703	56473.4375
150200	149832.0625	129581.1172	71406.75
165200	164831.875	144565.6406	86340.07813
180200	179831.7031	159550.1875	101273.3906
195200	194831.5313	174534.7188	116206.7188
210200	209831.3438	189519.2656	131140.0469
225200	224831.1719	204503.7969	146073.3594
240200	239831	219488.3438	161006.6719
255200	254830.8125	234472.875	175940.0156

Table No. IV

Table No. IV depicts the behaviour of the profit w.r.t. revenue per unit uptime of the system (C_0) for different values of rate failure of main unit (λ). From the table, it is seen that the profit increases with increase in the values of C_0. Also, following conclusions can be drawn:

1. For λ = 0.000032, profit is > or = or < according as C_0 > or = or < INR 366.10, i.e. the revenue per unit uptime of the system (C_0) should not be less than INR 366 in order to get positive profit.
2. For λ = 0.0032, profit is > or = or < according as C_0 > or = or < INR 20464, i.e. the revenue per unit uptime of the system (C_0) should not be less than INR 20464 in order to get positive profit.
3. For λ = 0.32, profit is > or = or < according as C_0 > or = or < INR 78688, i.e. the revenue per unit uptime of system (C_0) should not be less than INR 78688 in order to get positive profit.

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