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Influence of Crane Beams Mass Loss on Their Dynamic Behavior

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Abstract: Dynamic properties of driving beams overhead cranes ore, as for the rod with variable in time and along the span of the mass and stiffness of the cross sections. For solving differential equations of transverse vibrations are used Fourier series and Bessel functions.
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1. Introduction

It is quite an urgent problem nowadays to study and calculate the dynamic strength of metalwork elements, to determine qualitative and quantitative laws which stipulate the design service life.

Unlike many modern machines and equipment, obsolescence of load-lifting cranes, metal structures goes more slowly. To replace cranes, especially those with specific technical features and which are extremely metal-intensive designs is a costly business. Extension of crane working life leads to resources saving which is equivalent, under severe conditions, to production costs of similar new cranes. Therefore, service life prolongation makes up a significant reserve for saving money, materials, energy and labor costs. Residual life prediction provides with additional opportunities to save money and gain economic effect.

2. Problem Statement

Cranes often work in aggressive environment causing intensive corrosion of carrying metal which can cause accidents and disasters. For example, ore clamshell load transfer cranes that work in the ore stock yard of "Zaporozhstal" factory, are, on the one hand, influenced by coke oven emissions, and, on the other hand, – by blast furnace emissions. This circumstance leads to the fact that corrosion rate along the length of main beams is nonequal and increases according to certain laws towards supports located close to coke ovens. Besides, crane metal constructions are subject to intensive dynamic loads from the movement of carts in a loaded state, the weight of which can reach hundreds of tons, as well as unsteady wind loads. Intense long-term corrosion wear, variable moving loads, wind pressure and ripple all affect the dynamic characteristics of heavy crane metal structures.

This paper has made an attempt to evaluate the impact of significant corrosion damage on vibration characteristics of the main beam as the main carrying element of the metal structure of a bridge-type crane.

We will generally examine small transverse oscillations of the main beam of the crane as a rod with a rectilinear axis of variable in length and time section. Rod axis is the line of centers of gravity of its sections. The rod is supposed to have at least one longitudinal plane of symmetry and oscillations are considered in this plane.

Let us take the rectilinear axis of the main beam in its undeformed state for the axis x. Due to the smallness of oscillations, the displacement of any point of the axis at the time t coordinate $y\ (x,t)$, while the section rotation is determined by the angle $\theta = \dfrac{dy}{dx}$. We will neglect small displacements of the cross sections along the axis and the associated inertia forces.

Oscillations occur with small amplitudes, while maintaining proportionality between the elastic forces and the deviations of the rod from the equilibrium position or, which is the same thing, between the elastic forces and the rod deformations. Since the main beam has a variable mass, the equation of its oscillations should include summands containing the derivative of mass. However, due to the fact that corrosion processes develop relatively slowly, the rate of change of mass can be neglected. Under the assumptions made, the differential equation of free oscillations of the rod is as follows

$$\frac{d^2}{dx^2}\left(EJ\frac{d^2y}{dx^2}\right)-\frac{d}{dx}\left(J_s\frac{d^2\theta}{dt^2}+N\frac{dy}{dx}\right)+m\frac{d^2y}{dt^2}=0, \tag{1}$$

where $m(x,t), EJ(x,t)$ is linear mass of the rod and bending rigidity;

$J_s(x,t)$ is mass linear moment of the rod inertia.

With small deviations from the equilibrium position, the longitudinal forces N do not depend on oscillations and are completely determined by the initial statements of the problem.

The calculation of the frequencies of free oscillations of rods is usually carried out without considering inertia of rotation of its elements. This assumption, which is correct for lower oscillation modes, becomes unfair when calculating the frequencies of higher oscillation modes and if extraneous masses, which significantly change the mass linear moment of inertia, but hardly change its rigidity, are attached to the rod.

Nevertheless, at the beginning of the study, we will not take into account the rotation inertia of elements of the rod cross section and will assume that the longitudinal force N is equal to zero. In this case, the differential equation (1) is as follows:

$$\frac{d^2}{dx^2}\left(EJ\frac{d^2y}{dx^2}\right)+m\frac{d^2y}{dt^2}=0. \tag{2}$$

To solve the differential equation (2), we will apply the Fourier method in the case when the bending rigidity of the rod EJ and linear mass m are the product of functions $\psi(t)$, that depend only on time and functions $\varphi(\xi)$, that depend only on the coordinate of the section x

$$EJ=\varphi_1(\xi)\cdot\psi_1(t); \qquad m=\varphi_2(\xi)\cdot\psi_2(t), \tag{3}$$

where $\xi=\frac{x}{l}$ is dimensionless coordinate changing along the length of the rod from zero to one;

l is the length of the rod.

Let us represent the desired bending function $y(\xi,t)$ as the product of two independent functions

$$y(\xi_1,t)=Y(\xi)\cdot T(t). \tag{4}$$

Substitution (4), taking into account dependencies (3), into differential equation (2) leads to the separation of the latter into two independent equations

$$\ddot{T}(t)+p^2\frac{\psi_1(t)}{\psi_2(t)}T(t)=0, \tag{5}$$

$$\varphi_1(\xi)\cdot y^{(4)}+2\varphi_1'(\xi)\cdot y^{(3)}+\varphi_1''(\xi)\cdot y^{(2)}-l^4p^2\varphi_2(\xi)y=0. \tag{6}$$

Consider the case when the rod has a rectangular cross section, the linear dimensions of which decrease exponentially over time due to corrosion. The existence of an additional source that causes corrosion near one of the ends of the rod leads to the fact that along the length of the rod, the corrosion intensity is uneven and the linear dimensions of the cross section of the rod decrease exponentially as well as along the length of the rod.

First, let us consider the case when the height of the cross section is much larger than its width, so that a change in height can be neglected. As for a rectangular section

$$J = \frac{bh^3}{12}, \qquad m = \rho bh,$$

where b is rod cross section width;
h is section height.

Under the accepted assumption, denoting EJ_0 and m_0 as the initial bending rigidity and linear mass, we obtain the following dependences

$$EJ = EJ_0 e^{-\alpha(\tau+t)\xi} e^{-\beta(\tau+t)}, \tag{7}$$

$$m = m_0 e^{-\alpha(\tau+t)\xi} e^{-\beta(\tau+t)}. \tag{8}$$

Herein the following notations have been accepted:

τ is the lifetime of the rod (crane) in an aggressive environment before starting work within a particular cycle;
t is the current operating time of the rod (crane) within a particular cycle;
α и β are experimentally determined parameters.

Considering the fact that τ is significantly more than t, (for a crane τ is measured in decades, and t – in hours) the value t can be neglected in one of the factors in the dependencies (7) and (8).

We come to

$$EJ = EJ_0 e^{-\alpha\tau\xi} e^{-\beta(\tau+t)}, \tag{9}$$

$$m = m_0 e^{-\alpha\tau\xi} e^{-\beta(\tau+t)} \tag{10}$$

In this case, differential equations (5) and (6), which describe the free rod oscillations, will take the form of:

$$\ddot{T}(t) + p^2 T(t) = 0, \tag{11}$$

$$y^{(4)} - 2\alpha\tau y^{(3)} + \alpha^2\tau^2 y^{(2)} - k^4 y = 0, \tag{12}$$

where

$$k^4 = l^4 \frac{m_0 p^2}{EJ_0}. \tag{13}$$

This expression, solved relatively p, gives the general formula for calculating the frequency of natural oscillations of rods

$$p = \frac{k^2}{l^2}\sqrt{\frac{EJ_0}{m_0}}, \tag{14}$$

where coefficient k depends on conditions for fixing rods.

Equation (11) of free harmonic oscillations has a general solution

$$T(t) = A_1 \cos pt + A_2 \sin pt.$$

The general integral of equation (12) has the form

$$y(\xi)=c_1+c_2\xi+c_3e^{\alpha\tau\xi}+c_4\xi\cdot e^{\alpha\tau\xi},\quad \text{when } k^2=0, \tag{15}$$

$$y(\xi)=e^{\frac{\alpha\tau\xi}{2}}\left(c_1e^{\lambda_1\xi}+c_2e^{-\lambda_1\xi}+c_3e^{\lambda_2\xi}+c_4e^{-\lambda_2\xi}\right),\quad \text{when } 0<k^2<\frac{\alpha^2\tau^2}{4}, \tag{16}$$

$$y(\xi)=e^{\frac{\alpha\tau\xi}{2}}\left(c_1e^{\lambda_1\xi}+c_2e^{-\lambda_1\xi}+c_3\xi+c_4\right),\quad \text{when } k^2=\frac{\alpha^2\tau^2}{4}, \tag{17}$$

$$y(\xi)=e^{\frac{\alpha\tau\xi}{2}}\left(c_1e^{\lambda_1\xi}+c_2e^{-\lambda_1\xi}+c_3\cos\lambda_2\xi+c_4\sin\lambda_2\xi\right),\quad \text{when } k^2>\frac{\alpha^2\tau^2}{4}. \tag{18}$$

Thus,

$$\lambda_1=\sqrt{\frac{\alpha^2\tau^2}{4}+k^2},\qquad \lambda_2=\sqrt{\left|\frac{\alpha^2\tau^2}{4}-k^2\right|}. \tag{19}$$

The final view of the solution is obtained after determining the constants c_1,c_2,c_3,c_4, based on satisfying the boundary conditions for fixing the ends of the rod.

3. Let us consider a rod whose ends are located on rigid articulated supports, and the width $b(1,0)$ with $t=0$ of the end section for $\xi=1$ is 0.8 or 0.6 of the width $b(0,0)$ of the end section for $\xi=0$, i.e. $e^{-\alpha\tau}=0,8$ or $0,6(\alpha\tau=0,22;0,52)$ due to an additional source causing corrosion, and located near the end $\xi=1$.

The boundary conditions for the articulated supports are

$$y(0)=0,\qquad y''(0)=0,\qquad y(1)=0,\qquad y''(1)=0. \tag{20}$$

The task is to determine a non-zero solution of differential equation (12) satisfying the boundary conditions (20).

From satisfying boundary conditions (20) it follows that solutions (15), (16), (17) represent one solution $y(\xi)=0,$ since boundary conditions are satisfied by the only values of the unknown constants $c_1=c_2=c_3=c_4=0$ for the cases under consideration.

Substituting solution (18) into the boundary conditions (20), we obtain a homogeneous system of equations of relative unknowns c_1,c_2,c_3,c_4. To obtain non-zero solutions of the system, we equate its determinant to zero and obtain the frequency equation in the form of a fourth-order determinant. Expanding the determinant in a row, we come to the equation for determining frequencies in the form

$$\begin{aligned}&\sin\lambda_2\left(e^{-\lambda_1}-e^{\lambda_1}\right)\left[2\alpha\tau\lambda_1\left(\alpha\tau\lambda_1+2k^2\right)-\left(\alpha\tau\lambda_1+2k^2\right)^2+\alpha^2\tau^2\lambda_2^2\right]+\\&+2\alpha^2\tau^2\lambda_1\lambda_2\cos\lambda_2\left(e^{-\lambda_1}+e^{\lambda_1}\right)-4\alpha^2\tau^2\lambda_1\lambda_2=0.\end{aligned} \tag{21}$$

The roots of this equation, corresponding to the first tone for the lowest form of oscillations, produce the following frequency formulas

$$p=\frac{9,8434}{l^2}\sqrt{\frac{EJ_0}{m_0}}\qquad \text{for } \alpha\tau=0,52;\ e^{-\alpha\tau}=0,6. \tag{22}$$

$$p=\frac{9{,}7877}{l^2}\sqrt{\frac{EJ_0}{m_0}} \quad \text{for } \alpha\tau=0{,}92;\ e^{-\alpha\tau}=0{,}4. \tag{23}$$

If we compare the obtained results with a similar frequency for a rod of uniform section, whose coefficient in the frequency formula (14) in case of pivotally supported ends of the rod is equal to $k^2=\pi^2=9{,}8696...$, it can be seen that reducing the width of the cross section from one end of the rod to the other to $60(40)\%$ slightly affects the value of the frequency coefficient $0{,}83(0{,}25)\%$. Let us consider a cantilever rod with one end rigidly sealed $\xi=0$. The boundary conditions in this case are expressed by equations

$$y(0)=0;\ \ y'(0)=0;\ \ y''(1)=0;\ \ y'''(1)=0. \tag{24}$$

As it is in the previous case, solutions (15), (16), (17) of the differential equation (12), satisfying the boundary conditions (24), represent one solution $y(\xi)=0.$ The solution (18) in this case can be written in the following form

$$y(\xi)=e^{\frac{\alpha\tau\xi}{2}}\left(c_1 ch\lambda_1\xi+c_2 sh\lambda_1\xi+c_3\cos\lambda_2\xi+c_4\sin\lambda_2\xi\right), \tag{25}$$

where λ_1 and λ_2 are determined by (19), and $4k^2>\alpha^2\tau^2.$

Substituting boundary conditions (24) into solution (25), we obtain a homogeneous system of equations of relative unknowns $c_1,c_2,c_3,c_4.$ To obtain non-zero solutions of the system, we equate its determinant to zero and obtain the frequency equation which we bring to the form

$$(a_{11}-b_{11})(\lambda_2 a_{22}-\lambda_1 b_{22})-(a_{21}-b_{21})(\lambda_2 a_{12}-\lambda_1 b_{12})=0, \tag{26}$$

where

$$a_{11}=\left(\frac{\alpha^2\tau^2}{4}+r_1^2\right)chr_1+\alpha\tau r_1 shr_1=f_1\left(r_1,r_1^2,chr_1,shr_1\right),$$

$$a_{21}=\left(\frac{\alpha^3\tau^3}{8}+3\frac{\alpha\tau}{2}r_1^2\right)chr_1+\left(3\frac{\alpha^2\tau^2}{4}r_1+r_1^3\right)shr_1=f_2\left(r_1,r_1^2,r_1^3,chr_1,shr_1\right),$$

$$a_{12}=f_1\left(r_1,r_1^2,shr_1,chr_1\right),\ \ a_{22}=f_2\left(r_1,r_1^2,r_1^3,shr_1,chr_1\right),$$

$$b_{11}=f_1\left(-r_2,-r_2^2,\cos r_2,\sin r_2\right);\ \ b_{21}=f_2\left(-r_2,-r_2^2,r_2^3,\cos r_2,\sin r_2\right),$$

$$b_{12}=f_1\left(r_2,-r_2^2,\sin r_2,\cos r_2\right);\ \ b_{22}=f_2\left(r_2,-r_2^2,-r_2^3,\sin r_2,\cos r_2\right);$$

$$r_1=\lambda_1, r_2=\lambda_2.$$

Determining the roots of equation (26) that correspond to the first form of oscillations, we obtain the following frequency formulas

$$p=\frac{4{,}6265}{l^2}\sqrt{\frac{EJ_0}{m_0}} \quad \text{for } \alpha\tau=0{,}92,\ e^{-\alpha\tau}=0{,}4;$$

$$p = \frac{4{,}1138}{l^2}\sqrt{\frac{EJ_0}{m_0}} \quad \text{for } \alpha\tau = 0{,}52,\ e^{-\alpha\tau} = 0{,}6;$$

$$p = \frac{3{,}7597}{l^2}\sqrt{\frac{EJ_0}{m_0}} \quad \text{for } \alpha\tau = 0{,}22,\ e^{-\alpha\tau} = 0{,}8.$$

Comparing the obtained results with the frequency coefficient $k^2 = 3{,}5156$ for a similar oscillation form of the cantilever rod of uniform section, we can see that reduction of the cross section width from the sealed end to the free end up to 40% leads to an increase in the frequency coefficient by 17%.

Let us now consider the case when all linear dimensions of the cross section decrease exponentially over time and with a change in length, and therefore, the change in mass of a length unit and the section inertia moment is determined by the following dependences similar to (9), (10)

$$EJ = EJ_0 e^{-4\alpha\tau\xi} e^{-4\beta(\tau+t)}, \tag{27}$$

$$m = m_0 e^{-2\alpha\tau\xi} e^{-2\beta(\tau+t)}. \tag{28}$$

In this case differential equations (5) and (6) will look as follows:

$$\ddot{T}(t) + p^2 e^{-2\beta(\tau+t)} T(t) = 0, \tag{29}$$

$$y^{(4)} - 8\alpha\tau y^{(3)} + 16\alpha^2\tau^2 y^{(2)} - k^4 e^{2\alpha\tau\xi} y = 0. \tag{30}$$

Using substitution $\theta = \frac{1}{\beta} p e^{-\beta(\tau+t)}$, equation (29) is reduced to the Bessel equation and the general integral of equation (29) is expressed in terms of the zero-order Bessel functions

$$T(t) = A_1 J_0(\theta) + A_2 N_0(\theta). \tag{31}$$

The general integral of equation (30) is as follows:

$$y(\xi) = u^4 [c_1 J_4(u) + c_2 N_4(u) + c_3 J_4(u) + c_4 k_4(u)], \tag{32}$$

where

$$u = \frac{2k}{\alpha\tau} e^{\frac{\alpha\tau\xi}{2}}.$$

Since τ is much larger than t, the equation (29) can be simplified by neglecting the value t in the exponent with e, in comparison to τ. Equation (29) takes the form of the usual equation of free harmonic oscillations

$$\ddot{T}(t) + p^2 e^{-2\beta\tau} T(t) = 0, \tag{33}$$

whose general solution is

$$T(t) = A_1 \cos e^{-\beta\tau} pt + A_2 \sin e^{-\beta\tau} pt. \tag{34}$$

In the case when the parameter values are such that the product $2\alpha\tau\xi$ is small, we will expand $e^{2\alpha\tau\xi}$ in a row and keep the first three incomplete members of the row, by replacing ξ with an average value, thus, as first approximation, equation (30) takes the form similar to equation (12), the solution of which we have already considered:

$$y^{(4)} - 8\alpha\tau y^{(3)} + 16\alpha^2\tau^2 y^{(2)} - k^4\left(1 + \frac{\alpha\tau}{2}\right)^2 y = 0. \tag{35}$$

Having carried out similar transformations for the cantilever rod, for example, we get the following, correspondent to the first form of oscillations, approximate formulas of the frequency factor in (34)

$$p = \frac{5{,}079}{l^2}\sqrt{\frac{EJ_0}{m_0}}, \qquad \text{for } \alpha\tau = 0{,}52;\ e^{-\alpha\tau} = 0{,}6;$$

$$p = \frac{4{,}120}{l^2}\sqrt{\frac{EJ_0}{m_0}}, \qquad \text{for } \alpha\tau = 0{,}22;\ e^{-\alpha\tau} = 0{,}8.$$

It can be suggested that the linear dimensions of the cross section of the rod, due to corrosion, over time vary in inverse proportion to the time the rod was in an aggressive environment. The existence of an additional source that causes corrosion near one of the ends of the rod leads to the fact that along the length of the rod the corrosion intensity is non-uniform, whereupon the linear dimensions of the cross section also decrease along the length of the rod according to the parabolic law. In this case, for the length unit mass and the rod inertia moment we will obtain the following dependences

$$EJ = EJ_0 \frac{[\alpha(\tau + t)\xi - 1]^8}{[\beta(\tau + t) + 1]^4}, \tag{36}$$

$$m = m_0 \frac{[\alpha(\tau + t)\xi - 1]^4}{[\beta(\tau + t) + 1]^2}. \tag{37}$$

Taking into account the fact that $\tau \rangle\rangle t$, we neglect summand $\alpha t\xi$ in comparison to $\alpha\tau\xi$ and come to simpler expressions

$$EJ = EJ_0 \frac{(\alpha\tau\xi - 1)^8}{[\beta(\tau + t) + 1]^4}, \tag{38}$$

$$m = m_0 \frac{(\alpha\tau\xi - 1)^4}{[\beta(\tau + t) + 1]^2}. \tag{39}$$

Taking into account dependences (38), (39), differential equations (5) and (6), which describe free oscillations of the rod, will take the form:

$$\ddot{T}(t) + p^2 \frac{1}{(\beta\tau + \beta t + 1)^2} T(t) = 0, \qquad \beta \neq 0; \tag{40}$$

$$(\alpha\tau\xi - 1)^4 y^{(4)} + 16\alpha\tau(\alpha\tau\xi - 1)^3 y^{(3)} + 56\alpha^2\tau^2(\alpha\tau\xi - 1)^2 y^{(2)} - k^4 y = 0, \qquad \alpha \neq 0. \tag{41}$$

Equation (40) is the Euler equation, the solution of which is

$$T(t)=A_1(\beta\tau+\beta t+1)^{\frac{1}{2}+\lambda}+A_2(\beta\tau+\beta t+1)^{\frac{1}{2}-\lambda}\qquad \text{when } 1-\frac{4p^2}{\beta^2}>0,$$

$$T(t)=A_1(\beta\tau+\beta t+1)^{\frac{1}{2}}+A_2(\beta\tau+\beta t+1)^{\frac{1}{2}}\ln(\beta\tau+\beta t+1)\qquad \text{when } 1-\frac{4p^2}{\beta^2}=0,$$

$$T(t)=A_1(\beta\tau+\beta t+1)^{\frac{1}{2}}\cos\lambda\ln(\beta\tau+\beta t+1)+A_2(\beta\tau+\beta t+1)^{\frac{1}{2}}\sin\lambda\ln(\beta\tau+\beta t+1)\ \text{when}$$
$$1-\frac{4p^2}{\beta^2}<0,$$

$$2\lambda=\sqrt{\left|1-\frac{4p^2}{\beta^2}\right|}.$$

The general integral of equation (41) is the following

$$y(\xi)=c_1\left|\xi-\frac{1}{\alpha\tau}\right|^{-\frac{5}{2}+S_1}+c_2\left|\xi-\frac{1}{\alpha\tau}\right|^{-\frac{5}{2}-S_1}+c_3\left|\xi-\frac{1}{\alpha\tau}\right|^{-\frac{5}{2}+S_2}+c_4\left|\xi-\frac{1}{\alpha\tau}\right|^{-\frac{5}{2}-S_2},$$

if

$$\sqrt{\frac{k^4}{\alpha^4\tau^4}+9}<3+\frac{25}{4};$$

$$y(\xi)=c_1\left|\xi-\frac{1}{\alpha\tau}\right|^{-\frac{5}{2}+S_1}+c_2\left|\xi-\frac{1}{\alpha\tau}\right|^{-\frac{5}{2}-S_1}+c_3\left|\xi-\frac{1}{\alpha\tau}\right|^{-\frac{5}{2}}+c_4\left|\xi-\frac{1}{\alpha\tau}\right|^{-\frac{5}{2}}\ln\left|\xi-\frac{1}{\alpha\tau}\right|,$$

if

$$\sqrt{\frac{k^4}{\alpha^4\tau^4}+9}=3+\frac{25}{4};$$

$$y(\xi)=c_1\left|\xi-\frac{1}{\alpha\tau}\right|^{-\frac{5}{2}+S_1}+c_2\left|\xi-\frac{1}{\alpha\tau}\right|^{-\frac{5}{2}-S_1}+c_3\left|\xi-\frac{1}{\alpha\tau}\right|^{-\frac{5}{2}}\cos\left[S_2\ln\left|\xi-\frac{1}{\alpha\tau}\right|\right]+c_4\left|\xi-\frac{1}{\alpha\tau}\right|^{-\frac{5}{2}}\sin\left[S_2\ln\left|\xi-\frac{1}{\alpha\tau}\right|\right],$$

if

$$\sqrt{\frac{k^4}{\alpha^4\tau^4}+9}>3+\frac{25}{4}.$$

The final view of the solution is obtained after determining the unknown constants, based on the satisfaction of the initial and boundary conditions.

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