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Theoretical Basis for Creating the Surface of a Toothed Operating Tool for Tillage

Zoia Shanina[1], Leonid Martovytskyi[2], Vasylii Glushko1[2], Olena Syvachuk[3]
[1]*PhD in Mathematics and Physics*
[2]*PhD in technical science*
[3]*Senior lecturer, Department of foreign languages*
Zaporizhzhia National Technical University
64 Zhukovskoho str., Zaporizhzhia, Ukraine, 69063

Abstract*:* Viewing soil as a medium with linear elastoviscous deformation, a mathematical model is proposed, which makes it possible to develop a toothed tool with such a shape of the working surface that satisfies the agrotechnical, technological and economic indicators when soil tilling.

The operating tool is made in the form of a block of teeth. The valley and the protrusion of the tooth in the horizontal plane are made along a logarithmic spiral, the protrusions are made in the form of the fourth-degree parabola.

The toothed tool equation is presented in the form of a combination of rotation, displacement and compression matrices.

Keywords: soil, valleys and protrusions of teeth, логарифмическаяспираль, параболачетвертогопорядка, linear elastoviscous deformation

The fertility of black earth soils, as the main economic resource of Ukraine, depends on the quality of their processing. It is known that loosening is the key criterion of soil tillage quality. The level of soil loosening depends both on the geometry of the operating tool and on the kinematics of its movement in the process of work. Studies have shown that the geometry of the operating tool and the kinematics of its movement affect the energy output of soil treatment, as well as the stress-strain state of the tillable soil and the tool during its operation. In order to meet the above requirements for soil treatment, research of the interaction of the operating tool with the soil was carried out. When doing so, the soil can be viewed as a medium with linear elastoviscous deformation. In the event of contact of linear-elastoviscous bodies, the issue of creating a mathematical model of the working body is reduced to solving problems by methods of elasticity theory based on the "correspondence principle".

As a test object, it is recommended to initially take anoperating tool for soil treatment, made in the form of a block of teeth.

The interaction of the toothed operating tool with the soil can be represented as the interaction of two bodies at the contact site described by the integral equation of the contact problem (1)

$$\int_{-a}^{a} P(t)\ln\frac{1}{|t-x|}\,dt = f(x), \qquad (1)$$

where $f(x)$ isfunction specified within interval $(-a,a)$ and depending on the shape of the operating tool in the horizontal plane and deformative constants of the operating tool and soil.

This function can be determined from the equation

$$f(x) = \frac{C - f_1(x) - f_2(x)}{\nu_1 + \nu_2}, \qquad (2)$$

where $f_1(x)$, $f_2(x)$ are functions describing the configuration of the operating tool and soil in the horizontal plane; C issome constant; ν_1, ν_2 are deformable constants of soil and the operating tool.

To solve the problem of interaction of a toothed operating tool with the soil, the following restrictions should be taken:

1. The soil is schematically represented as a half-plane, which leads to the following features of the geometry of the soil surface

$$f_1(x) = f_1'(x) = f_1''(x) = x. \tag{3}$$

2. The deformable constant of the operating tool ν_2 can be neglected $\nu_2 \to 0$, since the rigidity of the operating tool is several times higher in comparison with the soil. So, the modulus of elasticity of soils does not exceed $50-80\,МПа$. Whereas for metals the modulus of elasticity is $2 \cdot 10^5\,МПа$.

The deformable constants of two media can be written as

$$\nu_1 = \frac{2}{\pi \cdot E_{\partial 1}}\left(1-\mu_{b_1}\right), \quad \nu_2 = \frac{2}{\pi \cdot E_{\partial 2}}\left(1-\mu_{b_2}\right), \tag{4}$$

Where $E_{\partial 1}$ isdeformation modulus for the linearly deformable first medium (material of the working body – steel); $E_{\partial 2}$ isdeformation modulus for a linearly deformable second medium (soil); μ_{b1} islateral expansion coefficient of steel; μ_{b2} islateral soil expansion coefficient.

It is known from the elasticity theory that there is a relation between the shear modulus and the modulus of elasticity through Poisson's ratio

$$E = 2(1-\mu_b)G, \tag{5}$$

Therefore

$$\mu_b = \frac{E}{2G} - 1.$$

Using the "correspondence principle" for a linearly deformable medium, it is possible to write

$$\mu_{b2} = \frac{E_\partial}{2G_\partial}, \tag{6}$$

Where $E_\partial = E$ isdeformation modulus equal to elastic modulus.

Since $\nu_1 \to 0$, then ν_2 when substituting (6) in (4) will be equal to

$$\nu_2 = \frac{1-\mu_{b2}}{\pi \cdot G_\partial}. \tag{7}$$

The pressure distribution law for each contact area of the toothed operating tool depends on the shape of the area.When a toothed operating tool with wedge-shaped teeth acts on the soil (the equation of the wedge configuration $y_2 = f_2(x) = Ax$), the pressure distribution is expressed by the equation

$$P(x) = -\frac{P}{\pi a} \ln \frac{a - \sqrt{a^2 - x^2}}{|x|}, \tag{8}$$

where $P(x)$ is an unknown function within the interval $(-a, a)$, satisfying equation (1); a is contact half-width; P is the force applied to the operating tool.

The maximum pressure, equal to infinity, develops at the tip of the wedge at (Figure 1.c). According to the literature data [1, 4], the process of cracking in the soil depends on the nature of the pressure distribution in the contact area. Primary cracks occur at points of maximum pressure. In this case, the primary crack occurs at the tip of the wedge.

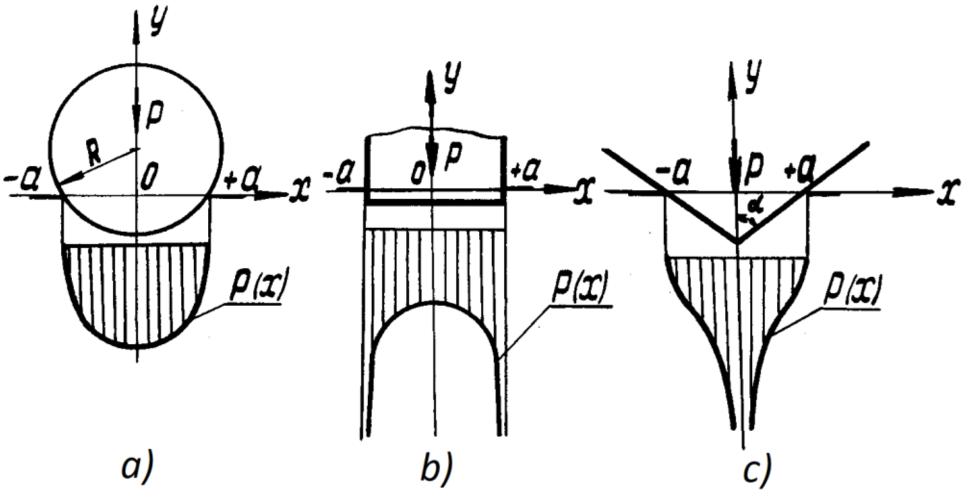


Figure 1 – Distribution of pressure of teeth of various shapes: a) round; b) flat; c) wedge-shaped.

In case of rectangular contact areas, a pressure concentration is created, causing a stress-strain state in a limited volume of soil. Thus, the pressure distribution is expressed by the equation

$$P(x) = \frac{P}{\pi\left(\sqrt{a^2 - x^2}\right)}. \quad (9)$$

Analysing equation (9), it is easy to see that the pressure at the edges of the rectangular contact areas of the toothed operating tool with $x = \pm a$ is maximum and tends to infinity (Figure 1.b). Cracks form and develop at the edges of the contact area.

The round shape of the protrusions of the toothed operating tool with radius R will have the following pressure distribution

$$P(x) = \frac{2P}{\pi \cdot a^2}\sqrt{a^2 - x^2}. \quad (10)$$

The maximum pressure develops at the point of initial contact, i.e. at $x = 0$,and equals

$$P(0) = \frac{1}{\pi}\sqrt{\frac{2\pi}{R \cdot v_2}}. \quad (11)$$

The circular contact section of the toothed operating tool, in comparison with the rectangular and wedge-shaped, has a more uniform pressure distribution (Figure 1.a).This pressure distribution causes the formation of several cracks with a more developed central one.

When various forms of teeth interact with the soil, due to the uneven distribution of contact pressures on the surface, the process of cracking does not occur over the entire contacting surface, and this affects the quality of soil treatment. Teeth made in the form of a wedge, a rectangle or a ball wear out during operation. Areas with maximum contact pressure are subject to intense wear. The wear process is not even. As a result, it is necessary to have such a shape of teeth that, when interacting with the soil, uniform pressures appear on the contact surface. This will entail evenly distributed cracking in the soil. This improves its loosening and reduces tool wear.

Suppose that the functions $f_1(x)$and $f_2(x)$, which determine the configuration of the operating tool and the soil in the horizontal plane, have continuous first and second derivatives. In the neighbourhood of point $x = 0$ (1)

$$f_1'(x) = f_2''(x) = 0. \quad (12)$$

Regarding the forces compressing the bodies, we will assume that their resultant perpendicular axes ox, are directed to the point of initial contact of the compressible bodies, i.e. to coordinate origin. Since the initial clearance between the compressible bodies $f_1(x)+f_2(x)$ is assumed to be symmetrical about axis oy, the pressure on the surfaces of the compressed bodies will also be symmetrical about axis oy.

Consider the case when the sum of the second derivatives is determined by the relation

$$f_1''(0)+f_2''(0)=0. \tag{13}$$

Suppose that not only the second derivative of the sum $f_1(x)+f_2(x)$, but all subsequent derivatives up to $2n-1$ inclusive, vanish at $x=0$. Meanwhile, derivative $f_1^{(2n)}(x)+f_2^{(2n)}(x)$ is nonzero at $x=0$, being continuous in this point. In this case, considering the smallness of the contact area with $-a \le x \le a$, we can approximately assume

$$f_1(x)+f_2(x)=\frac{1}{(2n)!}\left[f_1^{2n}(0)+f_2^{2n}(0)\right]\cdot x^{2n}. \tag{14}$$

Substituting (14) in (2) we find

$$f(x)=\alpha - A_n x^{2n}, \tag{15}$$

Where

$$A_n=\frac{f_1^{(2n)}(0)+f_2^{2n}(0)}{(2n)!(\nu_1+\nu_2)}. \tag{16}$$

The solution to the main integral equation (1) of the contact problem for the case when the right-hand side is represented in the form (15) is

$$P(x)=\frac{P}{\pi\cdot a^2}\sqrt{a^2-x^2}\cdot\left[\frac{2n}{2n-1}+\frac{2n(2n-2)}{(2n-1)(2n-3)}\cdot\frac{x^2}{a^2}+\frac{2n(2n-2)\ldots\cdot 2}{(2n-1)(2n-3)\ldots\cdot 3\cdot 2\cdot 1}\cdot\frac{x^{2n-2}}{a^{2n-2}}\right]. \tag{17}$$

We assume that a constant force P affects the operating tool, the half-width of the contact area a is known. Solving equation (17), we obtain the pressure distribution under the section of the toothed working body, the shape of which is a parabola of even degrees when changing n from 1 to 5. We obtain a uniform pressure distribution at $P(x)=const$. Using this ideal case, we will measure the actual pressure distributions by comparing the deviations of the obtained distributions with different n, in terms of the standard deviation from the mean (Figure 2).At the minimum value σ we obtain the optimal value n and the corresponding distribution $P(x)$ (Figure 3).

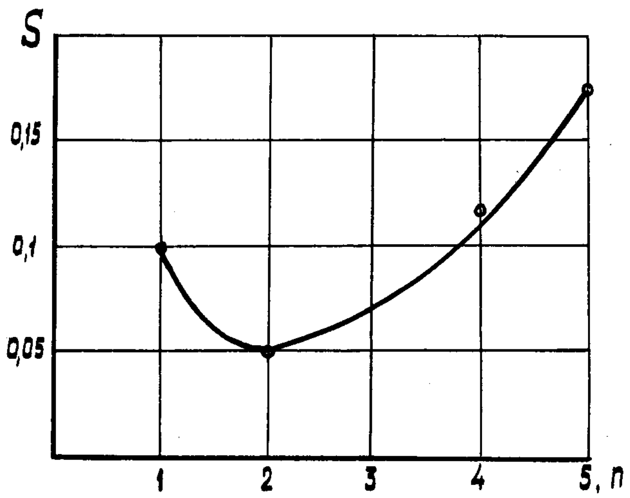
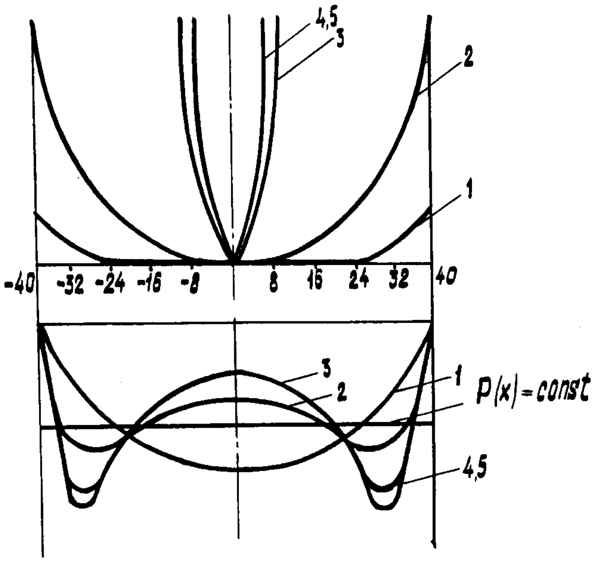


Figure 2 – Deviations from the mean depending on the exponent



Picture 3 – Distribution of tooth pressures depending on the exponent

The results of theoretical studies show that in order to obtain a more uniform distribution of contact pressures while the operating tool acts on the soil, the shape of the protrusions should be a fourth-degree parabola with the vertex directed towards the movement of the operating tool.In this case, the formation of cracks will occur more evenly over the entire contacting surface of the protrusions.The pressure on the contact area is as follows

$$P(x) = \frac{4P}{3\pi \cdot a^4}\left[\left(a^2 + 2x^2\right)\sqrt{a^2 - x^2}\right] . \qquad (18)$$

The task of designing a toothed operating tool for tillage machines is reduced to the choice of a mathematical model that would meet all the demands made to the operating tool in the technological, economic terms, considering the physical and mechanical properties of the soil.

The operating tool is made in the form of a block of teeth (Fig. 1) constituting valleys in the horizontal plane made along a logarithmic spiral, and protrusions profiled along the fourth-degree parabola.

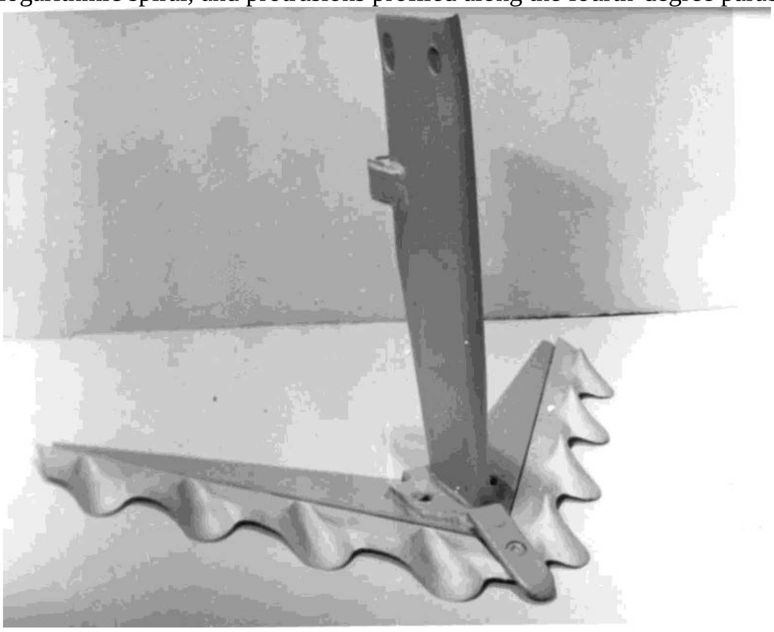


Figure 4 – General view of the toothed operating tool for tillage

In horizontally projecting planes, the section of the tooth is a family of logarithmic spirals

$$r_{\psi_i} = r_0 e^{\psi\, tg\varphi}, \qquad (19)$$

Where r_{ψ_i} is current radius vector; r_0 is initial radius vector; $tg\phi$ is internal friction coefficient; ψ is the current angle of the radius vector of the spiral.

The equation of the surface of the toothed operating tool for tillage can generally be represented using the matrix

$$A_{ij} = \left(\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ \hline 0 & 0 & 0 & 1 \end{array}\right), \qquad (20)$$

Where $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ is third-order matrix describing rotation in three-dimensional space; $(a_{14}, a_{24,}\, a_{34})^T$ are translational components.

The working surface will be set kinematically as trajectories of points of the generating logarithmic spiral (Fig. 5) located in the plane OX_2X_3 and performing rotational and translational movements.

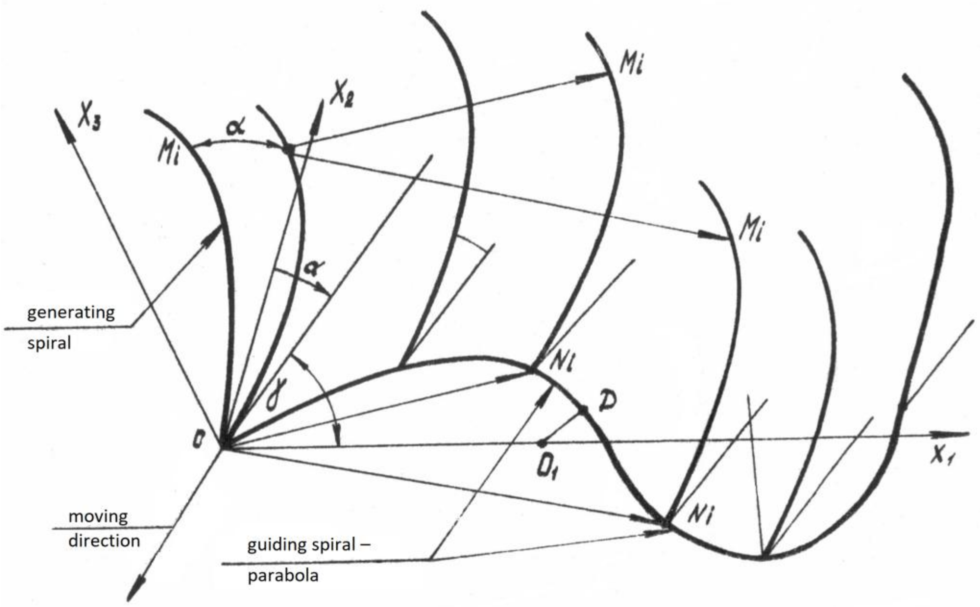


Figure 5 – Kinematic model of the surface of the operating tool

The rotation of the points M_i The rotation of the points OX_3 at the angle $\alpha = 90 - \gamma$ (where γ is the opening angle of the paw of the operating tool) is described by the rotation matrix

$$A_b = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \qquad (21)$$

The coordinates of the current points M_i of the generating logarithmic spiral, determined by equation (1), where $\varphi = 45^0$ and $\psi = -45^0 ... 50^0$ will have the following values (Fig. 6)

$$x_1^G = 0,\ x_2^G = r_{\psi_i} \sin\psi_i\ ; \quad x_3^G = r_{\psi_i} \cos\psi_i\,.$$

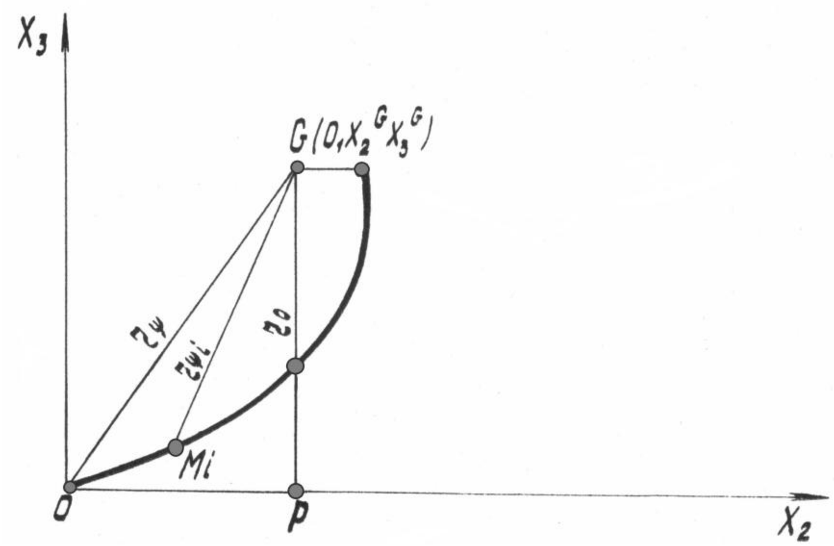


Figure 6 – Type of the generating spiral

The coordinates of the current points M_i of the generating spiral in Cartesian coordinates, depending on will be written as follows

$$x_1^{r\psi_i} = 0;$$
$$x_2^{r\psi_i} = OP - r_{\psi_i} \sin\psi_i; \qquad (22)$$
$$x_3^{r\psi_i} = GP - r_{\psi_i} \cos\psi_i.$$

The translational movement of points M_i in area of valley or protrusion is described by the displacement matrix.

$$A_n = \begin{pmatrix} 1 & 0 & 0 & \Delta x_1^{N_i} \\ 0 & 1 & 0 & \Delta x_2^{N_i} \\ 0 & 0 & 1 & \Delta x_3^{N_i} \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad (23)$$

Where $\Delta x_1^{N_i}$, $\Delta x_2^{N_i}$, $\Delta x_3^{N_i}$ are vector components.

Thus, the total displacement of the points of the generating logarithmic spiral is determined by scalar matrix multiplication

$$A = A_n \cdot A_b = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 & \Delta x_1^{N_i} \\ \sin\alpha & \cos\alpha & 0 & \Delta x_2^{N_i} \\ 0 & 0 & 1 & \Delta x_3^{N_i} \\ 0 & 0 & 0 & 1 \end{pmatrix}. \qquad (24)$$

Then the equation of the surface of the toothed operating tool can be represented as

$$Y_i = A \cdot X_i, \qquad (25)$$

Where X_i are coordinates of the current point M_i on the generating spiral, determined by the equations (22); Y_i are coordinates of points on the surface of the toothed operating tool depending on the parameter ψ_i, Which is the generating logarithmic spiral, and the magnitude of the vector ON_i determined by the position of the current point $N_i$on the valley or protrusion of the cutting blade edge.

For the valley area the coordinates of the current point $N_i$of the vector of displacement along a logarithmic spiral (Fig. 7) are determined by the following equations

$$x_1^{N_i} = x_1^{O_1} + r_{\theta_i} e^{\theta_i tg\varphi} \cos\theta_i;$$
$$x_2^{N_i} = r_{\theta_i} e^{\theta_i tg\varphi} \sin\bar{\theta}_i; \qquad (26)$$
$$x_3^{N_i} = 0,$$

Where θ_i =180° are initial angles; $\bar{\theta}_i = 20^o + \theta_i$ are coordinates of point $O_1\left(S/2, 0, 0\right)$; *S* is tooth pitch.

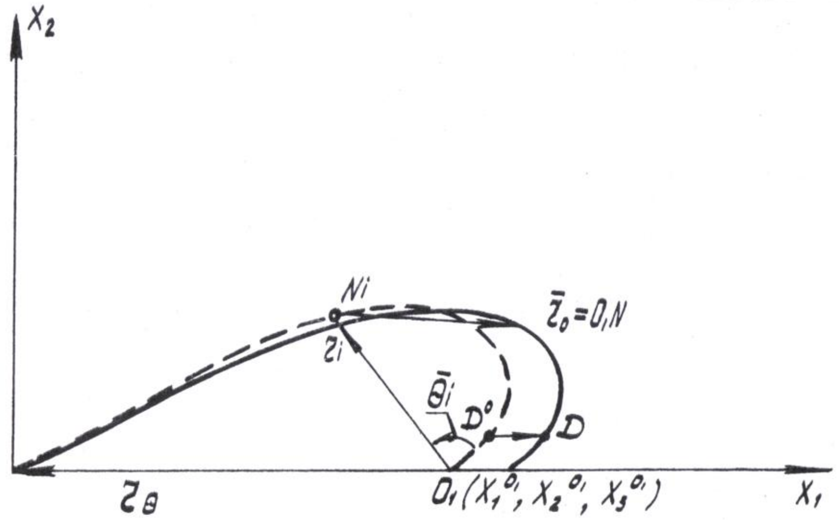


Figure 7 – Valley spiral transformations

In connection with the lengthening of the logarithmic spiral of the valley profile along the axis OX_1 the coefficient is introduced

$$\kappa = \frac{1}{\cos\alpha},$$

While the transformation of the elongation is written by the matrix

$$A_k^y = \begin{pmatrix} \kappa & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \qquad (27)$$

Considering equation (27) the coordinates of the points of the translational displacement vector can be determined as

$$\Delta x_i = A_k^y \cdot x_i;$$
$$\Delta x_1^{N_i} = \kappa x_1^{N_i}; \qquad (28)$$
$$\Delta x_2^{N_i} = x_2^{N_i};$$
$$\Delta x_3^{N_i} = 0.$$

For the protrusion area the coordinates of the current point N_i (Fig. 8) of the vector of translational displacement along the parabola $x_2 = x_1^4$ are determined as the product of the following transformations:

a) compression along the axis OX_2 with coefficient μ, described by the matrix

$$A_{m}^{c} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \quad (29)$$

b) elongation along the axis OX_1 with coefficient k, described by the matrix (27);

c) translational displacement along the vector CD, described by the matrix (23).

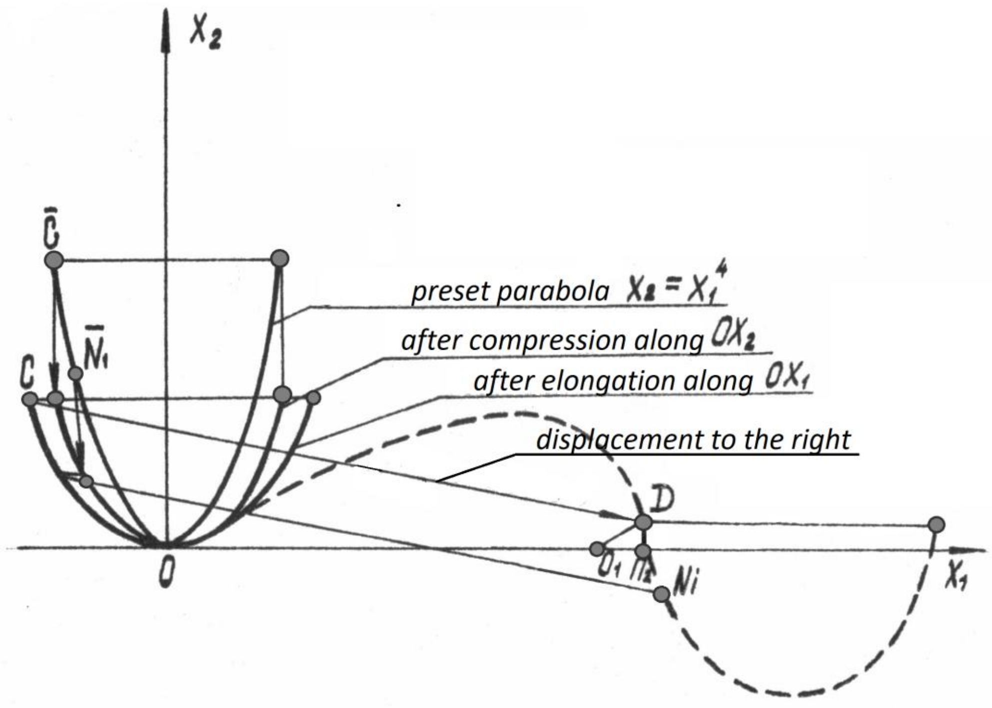


Figure 8 – Coordinates of the current point N_i of the parabola in the place of protrusions

The general transformation of the parabola is then described by the matrix

$$A_0 = A_n \cdot A_y \cdot A_l;$$

$$A_l = \begin{pmatrix} k & 0 & 0 & \Delta x_1^{CD} \\ 0 & \mu & 0 & \Delta x_2^{CD} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (30)$$

The coordinates of the current point N_i on the parabola will look like this in matrix form

$$\Delta x_i = A_0 \cdot X_i, \quad (31)$$

Where X_i are coordinates of points of the original parabola $x_2 = x_1^4$; Δx_i are coordinates of the points of the transformed parabola which are defined as follows:

$$\Delta x_1 = kx_1 + \Delta x_1^{CD};$$
$$\Delta x_2 = \mu x_2 + \Delta x_2^{CD}; \quad (32)$$
$$\Delta x_3 = 0.$$

The equation of the surface of the toothed operating body (25) in matrix form can be represented as

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 & \Delta x_1^{N_i} \\ \sin\alpha & \cos\alpha & 0 & \Delta x_2^{N_i} \\ 0 & 0 & 1 & \Delta x_3^{N_i} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1^{M_i} \\ x_2^{M_i} \\ x_3^{M_i} \\ 1 \end{pmatrix}, \qquad (33)$$

Where $x_1^{M_i}, x_2^{M_i}, x_3^{M_i}$ are coordinates of the points of the generating spiral; $\Delta x_1^{N_i}, \Delta x_2^{N_i}, \Delta x_3^{N_i}$ are coordinates of the points of the cutting blade.

This equation describes the surface of the tooth in the places of valleys and protrusions.

Finite formula for the coordinates of the tooth surface points for the valley area can be put down as follows

$$y_1 = \sin\alpha\left(OP - r_{\psi_i}\sin\psi_i\right) + k\left(S/2 + r_{\theta_i}\cos\bar{\theta}_i\right);$$

$$y_2 = \cos\alpha\left(GP - r_{\psi_i}\sin\psi_i\right) + r_{\theta_i}\sin\bar{\theta}_i; \qquad (34)$$

$$y_3 = GP - r_{\psi_i}\cos\psi_i,$$

And for the protrusion area in the following way

$$y_1 = \sin\alpha\left(OP - r_{\psi_i}\sin\psi_i\right) + kx_i + \Delta x_1^{CD};$$

$$y_2 = \cos\alpha\left(GP - r_{\psi_i}\sin\psi_i\right) + \mu x_2 + \Delta x_2^{CD}; \qquad (35)$$

$$y_3 = GP - r_{\psi_i}\cos\psi_i.$$

Conclusion

The obtained mathematical model (34), (35) for description of a toothed operating tool surface makes it possible, under various conditions, to obtain a family of toothed tillage tools, which will be workable in specific conditions, while taking into account the depth of tillage, agrotechnical requirements for the quality of crumbling soil, physical and mechanical properties of the soil, geometry of the cutting edge and the number of teeth. The shapes of the surfaces of the operating tools depend on the values of functional parameters k, μ, θ, ψ .

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