## Method of Determining Isotherm Coordinates in Continuous Casting of Steel

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**Abstract**: An example of solving a thermophysical problem is the problem of metal solidification in the process of continuous casting. Depending on the temperature value, the values of thermophysical parameters of steel change significantly in this interval. There arises a difficulty in solving such a problem, which involves the need to apply a fine grid throughout the solution area.

Presented is the method of calculating the isotherm coordinates during solidification of a continuous ingot based on difference relations. The proposed approach allows to increase the step in spatial and temporal coordinate, which significantly reduces the machine time and errors arising during the calculation. **Keywords:** The heat equation, the finite difference method, isotherms, isotherms coordinates.

There are a number of thermophysical problems for the analysis of which the main thing is the movement of the temperature front, in the zone of which phase transformations occur. One example of such problems can be the problem of crystallisation (solidification) of metal in the process of continuous casting on continuous casting machines. Steel solidification occurs in a relatively small crystallisation zone. Depending on the temperature value, the values of thermophysical parameters of steel change significantly in this interval.

The essential difference between these problems is the insignificant geometric dimensions of these zones, significant temperature gradients and the need for a detailed study of the temperature profile in these zones.

The difficulty arising in solving this kind of problems is the necessity to apply a fine grid to the whole solution area, or at least to the area adjacent to the previously unknown temperature front zone.

These difficulties can be substantially reduced by calculating at each time step the coordinates with a given temperature, i.e. finding isotherms with a given temperature step.

Let the function u(x,t) satisfy equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \tag{1}$$

With some boundary and initial conditions.

If u(x,t) is a solution of equation (1), then equation

$$u(x_i, t) = u_i = const$$
<sup>(2)</sup>

Is the implicit equation of i isotherm, i.e. it corresponds to the temperature  $u_i$ .

We differentiate (2) by t:

$$\frac{\partial u}{\partial x}\frac{dx_i}{dt} + \frac{\partial U}{\partial t} = 0 \qquad \qquad \frac{\partial u}{\partial x}\frac{dx_i}{dt} = -\frac{\partial^2 u}{\partial x^2}$$
(3)

Let  $x_{i-1}$  be the coordinate i-1 of the isotherm corresponding to temperature  $u_{i-1}$   $x_{i+1}$  are coordinates of the isotherm corresponding to temperature  $u_{i+1}$ . In turn, if we replace the derivatives by

difference relations:

$$\frac{\partial u}{\partial x} = \frac{u_{i+1} - u_{i-1}}{2(x_{i+1} - x_{i-1})}; \quad \frac{\partial^2 u}{\partial x^2} = \frac{\frac{u_{i+1} - u_i}{x_{i+1} - x_i} - \frac{u_i - u_{i-1}}{x_i - x_{i-1}}}{x_{i+1} - x_i}; \quad (4)$$

And substitute into (1), we obtain

$$\frac{u_{i+1} - u_{i-1}}{x_{i+1} - x_{i-1}} \cdot \frac{dx_i}{dt} = -2 \frac{\frac{u_{i+1} - u_i}{x_{i+1} - x_i} - \frac{u_i - u_{i-1}}{x_i - x_{i-1}}}{x_{i+1} - x_i}$$

Or

$$\frac{dx_i}{dt} = -\frac{2}{u_{i+1} - u_{i-1}} \left( \frac{u_{i+1} - u_i}{x_{i+1} - x_i} - \frac{u_i - u_{i-1}}{x_i - x_{i-1}} \right)$$
(5)

If isotherms are chosen with step  $h_u$  , then

$$\frac{dx_i}{dt} = -\frac{1}{h_u} \left( \frac{h_u}{x_{i+1} - x_i} - \frac{h_u}{x_i - x_{i-1}} \right)$$

Or

$$\frac{dx_i}{dt} = \frac{x_{i-1} - 2x_i + x_{i+1}}{(x_{i+1} - x_i)(x_i - x_{i-1})}$$
(6)

 $dx_i$ 

If dt is replaced by a difference relation:

$$\frac{x_{i,j+1} - x_{i,j}}{h_t} = \frac{x_{i-1,j} - 2x_{i,j} + x_{i+1,j}}{\left(x_{i+1,j} - x_{i,j}\right)\left(x_{i,j} - x_{i-1,j}\right)}$$

Or

$$x_{i,j+1} = x_{i,j} + l \frac{x_{i-1,j} - 2x_{i,j} + x_{i+1,j}}{(x_{i+1,j} - x_{i,j})(x_{i,j} - x_{i-1,j})}$$
(7)

There are a number of problems where it is necessary to fix only some isotherms, e.g., the problem of solidification and cooling of a continuous sheet ingot. In one-dimensional version it involves the heat conduction equation:

$$C_{M}(T)\rho\frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left[\lambda(T)\frac{\partial T}{\partial x}\right], \qquad x \in \left[-S,S\right]$$
<sup>(8)</sup>

The solution of equation (8) is carried out under the following boundary conditions:

$$\pm \lambda \frac{\partial T}{\partial x}\Big|_{no\theta} = \alpha \Big[T_{no\theta} - T_{cp}(t)\Big]$$
<sup>(9)</sup>

The initial temperature distribution in the ingot is described as a parabola of n degree:

$$T(x,0) = T_{och}^{o} - \left(T_{och}^{o} - T_{noe}^{o}\right) \left(\frac{x}{S}\right)^{n}$$

The symmetry condition is fulfilled on the ingot axis:

$$\left. \frac{\partial T}{\partial x} \right|_{x=0} = 0 \tag{11}$$

Let us point out isotherms  $T_{\pi-}$  liquidus temperature and  $T_{c-}$  solidus temperature, and on the section  $[T_{\pi}, T_{c}]$  we will build a grid with step  $h_{T}$  and find the coordinates of the selected isotherms using the

described above method. On  $[0, x_{\pi}]$  and  $[x_c, S]$  sections we will find the temperature by the grid method.

This approach allows to increase the grid step, and hence the time coordinate step, which allows to significantly reduce machine time and errors arising during calculations.

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(10)